

Preface

TAGSS (Trieste Algebraic Geometry Summer School) is an ongoing series of schools taught by outstanding women mathematicians, with the aim of bringing doctoral students, postdocs, and anyone interested, from a review of the basic constructions to current, state-of-the art research themes.

The previous successful editions featured programs on enumerative geometry (TAGSS I - Summer School in Enumerative Geometry, held at SISSA in 2017), geometry of moduli spaces of curves (TAGSS II - Summer School on Geometry of Moduli Spaces of Curves, held at ICTP in 2018), and applications of geometry to biochemical networks and data clouds (TAGSS III - Algebraic Geometry towards Applications, again at ICTP in 2019).

After a break in 2020 due to the pandemic situation, we reprised the series in July 2021 with the online event TAGSS 2021 - Hyperkähler and Prym Varieties: Classical and New Results, sponsored by ICTP (activity *smr 3609*, available at the page <https://indico.ictp.it/event/9610/overview>). It featured courses by Elham Izadi (Hyperkähler manifolds, an overview and some open problems) and Angela Ortega (Prym varieties). Following the same format as the previous editions, the Summer School lasted one week and included exercise sessions that complemented the lectures, as well as contributed talks delivered by young participants.

The topics of the school concerned the connection between Abelian varieties and algebraic curves, which has inspired algebraic geometers for more than a century, with each field helping to shed light on the other. With each (smooth projective) curve one can associate its Jacobian; however, most principally polarized abelian varieties cannot be obtained in this way. A more general construction associates with a finite morphism of curves its Prym variety; this construction leads to the concept of Prym-Tyurin variety. These associations work well in families, leading to the Torelli map from the moduli of curves (and the Prym map from the moduli of covers) to the moduli of principally polarized abelian varieties, and have led to the proof of numerous results on all these important moduli spaces. Other applications include Hodge theory, and in particular primal cohomology of the theta divisor, highlighting its connections to root lattices.

The main goal of the school was to provide a stimulating intellectual environment where all the participants could learn about some of these important

aspects of algebraic geometry, as well as the basic notions required for working in this field. In particular, we encouraged female students and researchers at the beginning of their career.

This volume contains the lecture notes of the two courses given at the school, as well as a contribution by one of the young speakers and her collaborator.

The lecture notes *Hyperkähler manifolds* contain contributions by Elham Izadi and some of her students and postdocs, who helped with the exercise sessions, namely Samir Canning, Yajnaseni Dutta, and David Stapleton. They give an elementary introduction to Hyperkähler manifolds, survey some of their interesting properties and some open problems.

The lecture notes *Prym varieties and Prym maps* have been co-authored by Angela Ortega and Paweł Borówka, who also led the exercise sessions of Ortega's course. They contain an introduction to the theory of Prym varieties, and a detailed analysis of the fibres of the Prym map for étale double coverings over genus 6 curves.

Finally, the article by Gian Paolo Grosselli and Irene Spelta concerns positive dimensional fibres of the Prym map $\mathcal{P}_{g,r}$. The authors present a direct procedure to investigate infinitely many examples of positive dimensional fibres.

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