Maple code for Lemma 3.5

```
> restart:
  Digits:=20:

  beta:=18*log(3)*log(1161/1000)/log(100/9):
  n0:=305494:
  g:=x->(x^2/2)*(log(x)+log(log(x))-3/2):
  B:=x->x*(log(x)+log(log(x))-0.9484):
  R:=x->g(x)*((x+1)/x*(g(x)/x)^(beta/(x*log(x)))-1):
  Lemma5:=x->R(x)-B(x+1):

  for n from 26*n0-1 by -1 while evalf(Lemma5(n))>0 do n od:
  print("The result of Lemma 5 is valid for n>="): n+1;
```

"The result of Lemma 5 is valid for n>="

30

Maple code for the lower bound in Theorem 1.1

```
> restart:
  Digits:=20:
  alpha:=2*log(2)*log(4/3)/log(5/2):
  S(0):=0:
  for k from 1 by 1 to 115149 do S(k):=S(k-1)+ithprime(k) od:
  Z:=n->S(n)*((n+1)/n*(S(n)/n)^(alpha/(n*log(n)))-1):
  for n from 115148 by -1 while evalf(ithprime(n+1)-Z(n))>10^(-18)
  do n od:
  print("The left hand side of Theorem is valid for n>="): n+1;
```

"The left hand side of Theorem is valid for n>="

3

Maple code for the upper bound in Theorem 1.1

```
> restart:
  Digits:=20:
  beta:=18*log(3)*log(1161/1000)/log(100/9):
  S(0):=0:
  for k from 1 by 1 to 305494 do S(k):=S(k-1)+ithprime(k) od:
  Z:=n->S(n)*((n+1)/n*(S(n)/n)^(beta/(n*log(n)))-1):
  for n from 305493 by -1 while evalf(Z(n)-ithprime(n+1))>10^(-18)
  do n od:
  print("The right hand side of Theorem is valid for n>="): n+1;
```

"The right hand side of Theorem is valid for n>="

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