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Perrin numbers expressible as sums of two base b repdigits

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Abstract. *In this paper we study Perrin numbers that can be expressed as sums of two base b repdigits. This can be done using linear forms in logarithms of algebraic numbers and a version of the Baker–Davenport reduction method.*

Keywords: Perrin sequence, linear forms in logarithms, repdigits, reduction method.
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1. Introduction

The Perrin sequence $\{P_n\}_{n\geq 0}$ is the ternary recurrent sequence defined as

$$P_{n+3} = P_{n+1} + P_n, \tag{1}$$

with initial terms $P_0 = 3, P_1 = 0$, and $P_2 = 2$. This is the sequence A001608 in the On-Line Encyclopedia of Integer Sequences (OEIS). The Perrin numbers are closely associated with the Padovan numbers (cf. A000931, OEIS) whose recurrence relation is same as that of Perrin sequence with different initials $(1, 0, 0)$. The first few Perrin numbers are

$$3, 0, 2, 3, 2, 5, 5, 7, 10, 12, 17, 22, 29, \ldots.$$

The closed form of Perrin sequence known as the Binet's formula is given by

$$P_n = \alpha^n + \beta^n + \gamma^n, \tag{2}$$

where α, β and γ are the roots of the characteristic equation $f(x) = x^3 - x - 1 = 0$ and they can be expressed in terms of radicals as

$$\alpha = \frac{r_1 + r_2}{6}, \quad \beta = \bar{\gamma} = \frac{-r_1 - r_2 + i\sqrt{3}(r_1 - r_2)}{12},$$

where $r_1 = \sqrt[3]{108+12\sqrt{69}}$ and $r_2 = \sqrt[3]{108-12\sqrt{69}}$. Numerically, the following estimates hold for α, β and γ :

$$\alpha \approx 1.32472, \quad |\beta| = |\gamma| \approx 0.868837.$$

One can observe that

$$|\beta| = |\gamma| = \alpha^{-1/2}.$$

The nth term of the Perrin sequence lies between α^{n-2} and α^{n+1} for all $n \geq 2$, that is,

$$\alpha^{n-2} \leq P_n \leq \alpha^{n+1}. \tag{3}$$

The above result can easily be shown by using induction.

For an integer $b \geq 2$, a natural number N of the form $N = a\left(\frac{b^l-1}{b-1}\right)$ for some $l \geq 2$ and $a \in \{1, 2, \ldots, b-1\}$ is called a base b repdigit. When $b = 10$, N is simply called a repdigit. Recently, many investigations have been made for searching repdigits in binary as well as ternary recurrent sequences. For example, Luca [11] proved that 55 and 11 are the largest repdigits in the Fibonacci and Lucas sequences respectively. Lucas, Pell and Pell-Lucas numbers as sums of two repdigits have been studied in [1, 2]. Rayaguru and Panda [13] searched the presence of repdigits in the product of consecutive balancing or Lucas-balancing numbers. In [14], they found all balancing and Lucas-balancing numbers which are expressible as sums of two repdigits. Bravo et al. [3] considered the Narayana's cows sequence (A000930 in the OEIS) and obtained all base b repdigits which are sum of two Narayana numbers. Lomelí and Hernández [10] determined all repdigits in Padovan sequence which can be written as sum of two Padovan numbers. In [7], Ddmulira found all repdigits which are sum of three Padovan numbers.

In this article we are interested to study Perrin numbers expressible as sums of two base b repdigits. More precisely, the exponential Diophantine equation

$$P_n = a_1\left(\frac{b^{l_1}-1}{b-1}\right) + a_2\left(\frac{b^{l_2}-1}{b-1}\right), \tag{4}$$

is to be solved in integers $2 \leq l_1 \leq l_2$ and $a_1, a_2 \in \{1, 2, \ldots, b-1\}$. The finiteness result can be easily deduced from the S-unit equation theorem, which was known since long ago. The contribution to the present work lies in the effectivity, in the sense that, in (4), n can be effectively bounded in terms of b. This can be achieved using Baker's method. There exist several different estimates of Baker-type lower bounds for linear forms in logarithms. In this study we use the most common Baker-type method due to Matveev ([12] or [4, Theorem 9.4]). This method also applies to every ternary linear recurrent sequence under mild assumptions, namely the existence of a dominant root.

Our main result is the following.

Theorem 1.1. *Let $b \geq 2$ be an integer. Then the Diophantine equation*

$$P_n = a_1\left(\frac{b^{l_1}-1}{b-1}\right) + a_2\left(\frac{b^{l_2}-1}{b-1}\right),$$

has only finitely many solutions in integers (n, a_1, a_2, l_1, l_2) *with* $1 \leq a_1, a_2 \leq b-1$ *and* $2 \leq l_1 \leq l_2$. *Moreover,* n *is bounded by* $1.35 \cdot 10^{31} \log^5 b$. *In particular, the only Perrin numbers expressible as sums of two repdigits are* $P_{11} = 22 = 11 + 11$ *and* $P_{20} = 277 = 55 + 222$.

2. Preliminary results

We need some results from Baker's theory of linear forms in logarithms of algebraic numbers for the proof of our main result. To start with, let η be an algebraic number with minimal polynomial

$$f(X) = a_0(X - \eta^{(1)}) \dots (X - \eta^{(k)}) \in \mathbb{Z}[X],$$

where $a_0 > 0$, and $(\eta^{(i)})_{1 \leq i \leq k}$ are the conjugates of η. Then,

$$h(\eta) = \frac{1}{k} \left(\log a_0 + \sum_{j=1}^{k} \max\{0, \log |\eta^{(j)}|\} \right),$$

is called the *logarithmic height* of η. In particular, if $\eta = a/b$ is a rational number with $\gcd(a, b) = 1$ and $b > 1$, then $h(\eta) = \log(\max\{|a|, b\})$. The following are some properties of logarithmic height function stated without special reference:

- $h(\eta + \gamma) \leq h(\eta) + h(\gamma) + \log 2$,
- $h(\eta\gamma^{\pm 1}) \leq h(\eta) + h(\gamma)$,
- $h(\eta^k) = |k| h(\eta)$.

With these notations, Matveev (see [12] or [4, Theorem 9.4]) proved the following result.

Theorem 2.1. *Let* $\mathbb{L}$ *be an algebraic number field of degree* $d_{\mathbb{L}}$.

Let $\eta_1, \eta_2, \dots, \eta_l \in \mathbb{L}$ *be positive real numbers and* $b_1, b_2, \dots, b_l$ *be non-zero integers. If* $\Gamma = \prod_{i=1}^{l} \eta_i^{b_i} - 1$ *is not zero, then*

$$\log |\Gamma| > -1.4 \cdot 30^{l+3} l^{4.5} d_{\mathbb{L}}^2 (1 + \log d_{\mathbb{L}})(1 + \log D) A_1 A_2 \dots A_l,$$

where $D \geq \max\{|b_1|, |b_2|, \dots, |b_l|\}$ *and* $A_1, A_2, \dots, A_l$ *are positive integers such that*

$$A_j \geq h'(\eta_j) = \max\{d_{\mathbb{L}} h(\eta_j), |\log \eta_j|, 0.16\} \quad \textit{for } j = 1, \dots, l.$$

We use the reduction method of Baker-Davenport due to Dujella and Pethő [8] for bound reduction. The following result will be used for reducing the bounds of the variables n, l_1, l_2 of (4).

LEMMA 2.2 ([8]). *Let M be a positive integer and p/q be a convergent of the continued fraction of the irrational number τ such that $\tau > 6M$. Let A, B, μ be some real numbers with $A > 0$ and $B > 1$. Let $\varepsilon := \|\mu q\| - M\|\tau q\|$, where $\|.\|$ denotes the distance from the nearest integer. If $\varepsilon > 0$, then there exists no solution to the inequality*

$$0 < |u\tau - v + \mu| < AB^{-w},$$

in positive integers u, v, w with

$$u \le M \text{ and } w \ge \frac{\log(Aq/\varepsilon)}{\log B}.$$

When $\mu = 0$, we get $\varepsilon < 0$. In this case we cannot apply Lemma 2.2. We use the following lemma due to Legendre.

LEMMA 2.3 (Legendre [5, 6]). *Let κ be a real number and x, y integers such that*

$$\left|\kappa - \frac{x}{y}\right| < \frac{1}{2y^2}.$$

Then $x/y = p_k/q_k$ is a convergent of κ. Further, let M and N be non-negative integers such that $q_N > M$. Then putting $a(M) = \max\{a_i : i = 0, 1, 2, \ldots, N\}$, the inequality

$$\left|\kappa - \frac{x}{y}\right| \ge \frac{1}{(a(M)+2)y^2},$$

holds for all pairs (x, y) of positive integers with $0 < y < M$.

The following result, needed in our proof, appears in [9].

LEMMA 2.4. *Let $r \ge 1$ and $H > 0$ be such that $H > (4r^2)^r$ and $H > L/(\log L)^r$. Then $L < 2^r H(\log H)^r$.*

The following result will be useful in proving our main result which gives a relation between n and l_2 of (4).

LEMMA 2.5. *All solutions of (4) satisfy*

$$(l_2 - 1)\log b - \log\alpha < n\log\alpha < l_2\log b + 2.$$

Proof. From (3), we have $\alpha^{n-2} \le P_n < 2 \cdot b^{l_2}$. Taking logarithm on both sides, we get

$$(n-2)\log\alpha < \log 2 + l_2\log b,$$

which leads to

$$n \log \alpha < l_2 \log b + 2.$$

Similarly, $b^{l_2-1} < P_n \leq \alpha^{n+1}$ gives

$$n \log \alpha > (l_2 - 1) \log b - \log \alpha.$$

This ends the proof. $\square$

3. Proof of Theorem 1.1

To start with, we find the upper bounds for the variables n, l_1, l_2 of (4). Using (2) in (4), we get

$$\alpha^n + \beta^n + \gamma^n = a_1 \left(\frac{b^{l_1} - 1}{b - 1} \right) + a_2 \left(\frac{b^{l_2} - 1}{b - 1} \right). \tag{5}$$

We examine (5) in two different steps. Firstly, we write (5) as

$$\alpha^n - \frac{a_2 b^{l_2}}{b-1} = \frac{a_1 b^{l_1}}{b-1} - \frac{a_1 + a_2}{b-1} - (\beta^n + \gamma^n).$$

Taking absolute values on both sides, we get

$$\left| \alpha^n - \frac{a_2 b^{l_2}}{b-1} \right| \leq \left| \frac{a_1 b^{l_1}}{b-1} \right| + \left| \frac{a_1 + a_2}{b-1} \right| + |\beta^n + \gamma^n| < 3 \cdot b^{l_1}.$$

Dividing both sides by $\frac{a_2 b^{l_2}}{b-1}$ implies

$$\left| \left(\frac{b-1}{a_2} \right) \alpha^n b^{-l_2} - 1 \right| < \frac{3}{b^{l_2 - l_1}} \cdot \frac{b-1}{a_2} < \frac{3}{b^{l_2 - l_1 - 1}}. \tag{6}$$

Put

$$\Gamma = \left(\frac{b-1}{a_2} \right) \alpha^n b^{-l_2} - 1. \tag{7}$$

We need to show $\Gamma \neq 0$. Suppose $\Gamma = 0$, then

$$\alpha^n = \frac{a_2}{b-1} b^{l_2}. \tag{8}$$

It is easily checked that α^n is irrational for every n, since β is not conjugate to α and $|\beta| \neq |\alpha|$. The irrationality of α immediately implies the non-vanishing of Γ. To apply Theorem 2.1 in (7), let

$$\eta_1 = \frac{b-1}{a_2},\ \eta_2 = \alpha,\ \eta_3 = b,\ b_1 = 1,\ b_2 = n,\ b_3 = -l_2,\ l = 3,$$

where $\eta_1,\ \eta_2,\ \eta_3\ \in \mathbb{Q}(\alpha)$ and $b_1,\ b_2,\ b_3 \in \mathbb{Z}$. The degree $d_{\mathbb{L}} = [\mathbb{Q}(\alpha) : \mathbb{Q}]$ is 3, where $\mathbb{L}$ is $\mathbb{Q}(\alpha)$.

Since $b^{l_2-1} < P_n < \alpha^{n+1}$, we have $l_2 < n$. Therefore, $D = \max\{1, n, l_2\} = n$. To estimate the parameters A_1, A_2, A_3, we calculate the logarithmic heights of η_1, η_2, η_3 as follows:

$$\begin{aligned} h(\eta_1) &= h\left(\frac{b-1}{a_2}\right) \le h(b-1) + h(a_2) \le 2\log(b-1) < 2\log b, \\ h(\eta_2) &= h(\alpha) = \frac{\log\alpha}{3} \text{ and } h(\eta_3) = h(b) = \log b. \end{aligned}$$

Thus, one can take

$$A_1 = 6\log b, \quad A_2 = \log\alpha \quad \text{and} \quad A_3 = 3\log b.$$

Then, we apply Theorem 2.1 and find

$$\log|\Gamma| > -1.4 \cdot 30^6 3^{4.5} 3^2 (1 + \log 3)(1 + \log n)(6\log b)(\log\alpha)(3\log b).$$

Comparing the above inequality with (6) gives

$$\begin{aligned} (l_2 - l_1 - 1)\log b &< \log 3 + 1.36 \cdot 10^{13}(1 + \log n)\log^2 b \\ &< 1.5 \cdot 10^{13}(1 + \log n)\log^2 b. \end{aligned}$$

Then, we get

$$(l_2 - l_1) < 1.6 \cdot 10^{13}(1 + \log n)\log b. \tag{9}$$

Secondly, we rewrite (5) as

$$\alpha^n - \frac{a_1 b^{l_1} + a_2 b^{l_2}}{b-1} = -\frac{a_1 + a_2}{b-1} - (\beta^n + \gamma^n),$$

which implies

$$\left|\alpha^n - \frac{a_1 b^{l_1} + a_2 b^{l_2}}{b-1}\right| \le \left|\frac{a_1 + a_2}{b-1}\right| + |\beta^n + \gamma^n| < 2.5.$$

Dividing both sides by α^n, we obtain

$$\left|1 - \alpha^{-n} b^{l_2}\left(\frac{a_1 b^{l_1 - l_2} + a_2}{b-1}\right)\right| < \frac{2.5}{\alpha^n}. \tag{10}$$

Put

$$\Gamma' = 1 - \alpha^{-n} b^{l_2}\left(\frac{a_1 b^{l_1 - l_2} + a_2}{b-1}\right).$$

Using similar arguments as above we can show that $\Gamma' \neq 0$. With the notations of Theorem 2.1, we take

$$\eta_1 = \alpha,\ \eta_2 = b,\ \eta_3 = \frac{a_1 b^{l_1-l_2} + a_2}{b-1},\ b_1 = -n,\ b_2 = l_2,\ b_3 = 1,\ l = 3,$$

where $\eta_1,\ \eta_2,\ \eta_3\ \in \mathbb{Q}(\alpha)$ and $b_1,\ b_2,\ b_3 \in \mathbb{Z}$. The degree $d_{\mathbb{L}} = [\mathbb{Q}(\alpha) : \mathbb{Q}]$ is 3, where $\mathbb{L} = \mathbb{Q}(\alpha)$.

Since $l_2 < n$, $D = n$. Computing the logarithmic heights of η_1, η_2 and η_3, we get

$$h(\eta_1) = \frac{\log \alpha}{3},\ h(\eta_2) = \log b$$

and

$$\begin{aligned} h(\eta_3) &\leq h(a_1 b^{l_1-l_2} + a_2) + h(b-1) \\ &\leq h(a_1) + (l_2 - l_1)\, h(b) + h(a_2) + h(b-1) + \log 2 \\ &< 3\log b + \log 2 + (l_2 - l_1) \log b \\ &\leq 4 \log b + (l_2 - l_1) \log b. \end{aligned}$$

Hence from (9), we get

$$h(\eta_3) < 4 \log b + 1.6 \cdot 10^{13}(1 + \log n) \log^2 b.$$

So, we take

$$A_1 = \log \alpha, \quad A_2 = 3 \log b \ \text{ and } \ A_3 = 4.9 \cdot 10^{13}(1 + \log n) \log^2 b.$$

Using all these values in Theorem 2.1, we have

$$\begin{aligned} \log |\Gamma'| > -1.4 \cdot 30^6 3^{4.5} 3^2 (1 + \log 3)(1 + \log n)(\log \alpha)(3 \log b) \\ \cdot (4.9 \cdot 10^{13}(1 + \log n) \log^2 b). \end{aligned}$$

Comparing the above inequality with (10) gives

$$n \log \alpha - \log 2.5 < 1.12 \cdot 10^{26}(1 + \log n)^2 \log^3 b. \tag{11}$$

Thus, we conclude that

$$n < 4.33 \cdot 10^{26}(1 + \log n)^2 \log^3 b. \tag{12}$$

With the notation of Lemma 2.4, we take $r = 2$, $L = n$ and $H = 4.33 \cdot 10^{26} \log^3 b$. Applying the lemma, we have

$$\begin{aligned} n &< 2^2(4.33 \cdot 10^{26} \log^3 b)(\log(4.33 \cdot 10^{26} \log^3 b))^2 \\ &< (17.32 \cdot 10^{26} \log^3 b)(62 + 3 \log\log b)^2 \\ &< (17.32 \cdot 10^{26} \log^3 b)(88 \log b)^2 \\ &< 1.35 \cdot 10^{31} \log^5 b. \end{aligned}$$

In the above inequality, we have used the fact that $62 + 3\log\log b < 88\log b$, which holds for all $b \geq 2$. Hence, we summarize that all the solutions of (4) satisfy

$$n < 1.35 \cdot 10^{31} \log^5 b.$$

Hence, the proof of Theorem 1.1 is finished. $\square$

REMARK 3.1. The above inequality allows one to compute all the solutions to (4) for every fixed b.

Now, as an illustration, we solve (4) for $b = 10$. When $b = 10$, the bound on n becomes

$$n < 8.74 \cdot 10^{32}.$$

Using Lemma 2.5, we get $l_1 \leq l_2 < 1.07 \cdot 10^{32}$.

Next, we reduce these bounds with the help of Lemma 2.2. Put

$$\Lambda = n\log\alpha - l_2\log 10 + \log\left(\frac{9}{a_2}\right). \tag{13}$$

The inequality (6) can be written as

$$\left|\left(\frac{9}{a_2}\right)\alpha^n 10^{-l_2} - 1\right| = |e^{\Lambda} - 1| < \frac{3}{10^{l_2 - l_1 - 1}}.$$

Observe that $\Lambda \neq 0$ as $e^{\Lambda} - 1 = \Gamma \neq 0$. Assuming $l_2 - 1_1 \geq 2$, the right-hand side in the above inequality is at most $\frac{3}{10} < \frac{1}{2}$. The inequality $|e^z - 1| < y$ for real values of z and y implies $z < 2y$. Thus, we get

$$|\Lambda| < \frac{6}{10^{l_2 - l_1 - 1}},$$

which implies that

$$\left|n\log\alpha - l_2\log 10 + \log\left(\frac{9}{a_2}\right)\right| < \frac{6}{10^{l_2 - l_1 - 1}}.$$

Dividing both sides by $\log 10$ gives

$$\left|n\left(\frac{\log\alpha}{\log 10}\right) - l_2 + \frac{\log(9/a_2)}{\log 10}\right| < \frac{3}{10^{l_2 - l_1 - 1}}. \tag{14}$$

To apply Lemma 2.2 in (14), let

$$u = n,\ \tau = \left(\frac{\log\alpha}{\log 10}\right),\ v = l_2,\ \mu = \frac{\log(9/a_2)}{\log 10},$$
$$A = 3,\ B = 10,\ w = l_2 - 1_1 - 1.$$

Choose $M = 8.74 \cdot 10^{32}$. We find q_{74} exceeds $6M$ with $0.009322 < \varepsilon = \|\mu q_{74}\| - M\|\tau q_{74}\|$. Applying Lemma 2.2 for $1 \le a_2 < 9$, we get $l_2 - l_1 \le 36$.

For the case $a_2 = 9$, we have that $\mu(a_2) = 0$. In this case we apply Lemma 2.3. The inequality (14) can be rewritten as

$$\left|\frac{\log \alpha}{\log 10} - \frac{l_2}{n}\right| < \frac{3}{n \cdot 10^{l_2 - l_1 - 1}} < \frac{1}{2n^2},$$

because $n < 8.74 \cdot 10^{32} = M$. It follows from Lemma 2.3 that $\frac{l_2}{n}$ is a convergent of $\kappa = \frac{\log \alpha}{\log 10}$. So $\frac{l_2}{n}$ is of the form p_k/q_k for some $k = 0, 1, 2, \ldots, 74$. Thus,

$$\frac{1}{(a(M)+2)n^2} \le \left|\frac{\log \alpha}{\log 10} - \frac{l_2}{n}\right| < \frac{3}{n \cdot 10^{l_2 - l_1 - 1}}.$$

Since $a(M) = \max\{a_k : k = 0, 1, 2, \ldots, 74\} = 49$, we get

$$l_2 - l_1 - 1 \le \frac{\log(3 \cdot (8.74 \cdot 10^{32}) \cdot 51)}{\log 10} < 35.12.$$

Thus $l_2 - l_1 \le 36$ in both cases.

Now for $1 \le a_1, a_2 \le 9$ and $l_2 - l_1 \le 36$, put

$$\Lambda' = -n \log \alpha + l_2 \log 10 + \log\left(\frac{a_1 10^{l_1 - l_2} + a_2}{9}\right). \tag{15}$$

From (5), we have

$$\alpha^n \left(1 - e^{\Lambda'}\right) = -\left(\frac{a_1 + a_2}{9}\right) - (\beta^n + \gamma^n).$$

Furthermore, we obtain

$$\frac{a_1 + a_2}{9} + (\beta^n + \gamma^n) > 0.$$

So $e^{\Lambda'} - 1 > 0$. Thus, $\Lambda' > 0$ and we have

$$0 < \Lambda' < e^{\Lambda'} - 1 = |\Gamma'| < \frac{2.5}{\alpha^n}.$$

This implies

$$\left|-n \log \alpha + l_2 \log 10 + \log\left(\frac{a_1 10^{l_1 - l_2} + a_2}{9}\right)\right| < \frac{2.5}{\alpha^n}.$$

Dividing both sides by $\log\alpha$ gives

$$\left| l_2\left(\frac{\log 10}{\log\alpha}\right) - n + \left(\frac{\log\left((a_1 10^{l_1-l_2}+a_2)/9\right)}{\log\alpha}\right)\right| < 9\cdot\alpha^{-n}. \tag{16}$$

Now, let

$$u = l_2,\ \tau = \left(\frac{\log 10}{\log\alpha}\right),\ v = n,\ \mu = \left(\frac{\log\left((a_1 10^{l_1-l_2}+a_2)/9\right)}{\log\alpha}\right),$$
$$A = 9,\ B = \alpha,\ w = n.$$

Choose $M = 8.74\cdot 10^{32}$. We find q_{83} exceeds $6M$ with $0.0000315 < \varepsilon = \|\mu q_{83}\| - M\|\tau q_{83}\|$. Then we apply Lemma 2.2 to the inequality (16) for $1 \le a_1, a_2 \le 9$ and $l_2 - l_1 \le 36$ and get $n \le 349$.

When $l_1 = l_2$ and $a_1 + a_2 = 9$, $\mu(a_1, a_2) = 0$. So, in this case we use Lemma 2.3. The inequality (16) can be rewritten as

$$\left|\frac{\log 10}{\log\alpha} - \frac{n}{l_2}\right| < \frac{9}{l_2\alpha^n} < \frac{1}{2l_2^2}.$$

Thus, we have

$$\frac{1}{(a(M)+2)l_2^2} \le \left|\frac{\log 10}{\log\alpha} - \frac{n}{l_2}\right| < \frac{9}{l_2\alpha^n}.$$

Since $a(M) = \max\{a_k : k = 0, 1, 2, \ldots, 83\} = 49$, we get

$$n \le \frac{\log(9\cdot(8.74\cdot 10^{32})\cdot 51)}{\log\alpha} < 292.$$

Thus $n \le 349$ in both cases.

We compute all the solutions of (4) using *Mathematica* for the range $n \le 349$ and find the following solutions,

$$P_{11} = 22 = 11 + 11 = \tfrac{10^2-1}{10-1} + \tfrac{10^2-1}{10-1},$$
$$P_{20} = 277 = 55 + 222 = 5\left(\tfrac{10^2-1}{10-1}\right) + 2\left(\tfrac{10^3-1}{10-1}\right).$$

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