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Weakly ωb-Continuous Functions[1]

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Abstract. *In this paper we introduce a new class of functions called weakly ωb-continuous functions and investigate several properties and characterizations. Connections with other existing concepts, such as ωb-continuous and weakly b-continuous functions, are also discussed.*

Keywords: b-Open Sets, ωb-Open Sets, Weakly b-Continuous Functions, Weakly ωb-Continuous Functions.
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1. Introduction

The notion of $b-open$ sets in topological spaces was introduced in 1996 by Andrijevic [1]. This type of sets discussed by El-Atik [2] under the name of $\gamma-open$ sets. In 2008, Noiri, Al-Omari and Noorani [4] introduced the notions of $\omega b-open$ sets and ωb-continuous functions. We continue to introduce and study properties and characterizations of weakly ωb-continuous functions.

Let A be a subset of a space (X, τ). The closure (resp. interior) of A will be denoted by $Cl(A)$ (resp. $Int(A)$).

A subset A of a space (X, τ) is called $b-open$ [1] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$. The complement of a $b-open$ set is called a $b-closed$ set. The union of all $b-open$ sets contained in A is called the $b-interior$ of A, denoted by $bInt(A)$ and the intersection of all $b-closed$ sets containing A is called the $b-closure$ of A, denoted by $bCl(A)$. The family of all $b-open$ (resp. $b-closed$) sets in (X, τ) is denoted by $BO(X)$ (resp. $BC(X)$).

Definition 1.1. *A subset A of a space X is said to be $\omega b-open$ [4] if for every $x \in A$, there exists a $b-open$ subset $U_x \subseteq X$ containing x such that $U_x - A$ is countable.*

The complement of an $\omega b-open$ set is said to be $\omega b-closed$ [4]. The intersection of all $\omega b-closed$ sets of X containing A is called the $\omega b-closure$ of A and is denoted by $\omega bCl(A)$. The union of all $\omega b-open$ sets of X contained in A is called the $\omega b-interior$ of A and is denoted by $\omega bInt(A)$.

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LEMMA 1.2 ([4]). *For a subset of a topological space, b-opennes implies $\omega b-$ openness.*

LEMMA 1.3 ([4]). *The intersection of an $\omega b-open$ set with an open set is $\omega b-open$.*

LEMMA 1.4 ([4]). *The union of any family of $\omega b-open$ sets is $\omega b-open$.*

2. Weakly ωb-Continuous Functions

DEFINITION 2.1. *A function $f:(X,\tau)\rightarrow(Y,\rho)$ is said to be:*

(a) ωb-continuous [4] if for each $x\in X$ and each open set V in Y containing $f(x)$, there exists an $\omega b-open$ set U in X containing x such that $f(U)\subseteq V$.

(b) weakly b-continuous [7] if for each $x\in X$ and each open set V in Y containing $f(x)$, there exists a b–open set U in X containing x such that $f(U)\subseteq Cl(V)$.

DEFINITION 2.2. *A function $f:(X,\tau)\rightarrow(Y,\rho)$ is said to be weakly ωb-continuous if for each $x\in X$ and each open set V in Y containing $f(x)$, there exists an $\omega b-open$ set U in X containing x such that $f(U)\subseteq Cl(V)$.*

REMARK 2.3. *Every ωb-continuous function is weakly ωb-continuous, but the converse is not true in general as the following example shows.*

EXAMPLE 2.4. *Let $X=R$ with the usual topology τ and $Y=\{a,b\}$ with $\rho=\{\phi,Y,\{a\}\}$. Define a function $f:(X,\tau)\rightarrow(Y,\rho)$ by $f(x)=a$ if $x\in Q$ and $f(x)=b$ if $x\in R-Q$. Then f is weakly ωb-continuous but not ωb-continuous.*

REMARK 2.5. *Since every $b-open$ set is $\omega b-open$ then every weakly b-continuous function is weakly ωb-continuous but the converse is not true in general as the following example shows.*

EXAMPLE 2.6. *Let $X=Y=\{a,b,c\}$, $\tau=\{\phi,X,\{c\},\{a,b\},\{a,b,c\}\}$ and $\rho=\{\phi,Y,\{a,b\},\{c,d\}\}$. Define a function $f:(X,\tau)\rightarrow(Y,\rho)$ by $f(a)=a$, $f(b)=d$, $f(c)=c$ and $f(d)=b$. Then f is weakly ωb-continuous but not weakly b-continuous.*

THEOREM 2.7. *A function $f:(X,\tau)\rightarrow(Y,\rho)$ is weakly ωb-continuous if and only if for every open set V in Y, $f^{-1}(V)\subseteq\omega bInt[f^{-1}(Cl(V))]$.*

Proof.

$\Rightarrow$) Let $V\in\rho$ and $x\in f^{-1}(V)$. Then there exists an $\omega b-open$ set U in X such that $x\in U$ and $f(U)\subseteq Cl(V)$. Therefore, we have $x\in U\subseteq f^{-1}(Cl(V))$ and hence $x\in\omega bInt[f^{-1}(Cl(V))]$ which means that $f^{-1}(V)\subseteq\omega bInt[f^{-1}(Cl(V))]$.

$\Leftarrow$) Let $x \in X$ and $V \in \rho$ with $f(x) \in V$. Then $x \in f^{-1}(V) \subseteq \omega bInt[f^{-1}(Cl(V))]$. Let $U = \omega bInt[f^{-1}(Cl(V))]$. Then U is $\omega b - open$ and $f(U) \subseteq Cl(V)$. □

THEOREM 2.8. *Let $f : (X, \tau) \to (Y, \rho)$ be a weakly ωb-continuous function. If V is a clopen subset of Y, then $f^{-1}(V)$ is $\omega b - open$ and $\omega b - closed$ in X.*

Proof. Let $x \in X$ and V be a clopen subset of Y such that $f(x) \in V$. Then there exists an $\omega b - open$ set U in X containing x such that $f(U) \subseteq Cl(V)$. Hence $x \in U$ and $f(U) \subseteq V$ and so $x \in U \subseteq f^{-1}(V)$. This shows that $f^{-1}(V)$ is $\omega b - open$ in X. Since $Y - V$ is a clopen set in Y, so $f^{-1}(Y - V)$ is $\omega b - open$ in X. But $f^{-1}(Y - V) = X - f^{-1}(V)$. Therefore $f^{-1}(V)$ is $\omega b - closed$ in X. Hence $f^{-1}(V)$ is $\omega b - open$ and $\omega b - closed$ in X. □

THEOREM 2.9. *A function $f : (X, \tau) \to (Y, \rho)$ is weakly ωb-continuous if and only if for every closed set C in Y, $\omega bCl[f^{-1}(Int(C))] \subseteq f^{-1}(C)$.*

Proof.

$\Rightarrow$) Let C be a closed set in Y. Then $Y - C$ is an open set in Y so by Theorem 2.8 $f^{-1}(Y - C) \subseteq \omega bInt[f^{-1}(Cl(Y - C))] = \omega bInt[f^{-1}(Y - Int(C))] = X - \omega bCl[f^{-1}(Int(C))]$. Thus $\omega bCl[f^{-1}(Int(C))] \subseteq f^{-1}(C)$.

$\Leftarrow$) Let $x \in X$ and $V \in \rho$ with $f(x) \in V$. So $Y - V$ is a closed set in Y. So by assumption $\omega bCl[f^{-1}(Int(Y - V))] \subseteq f^{-1}(Y - V)$. Thus $x \notin \omega bCl[f^{-1}(Int(Y - V))]$. Hence there exists an $\omega b - open$ set U in X such that $x \in U$ and $U \cap f^{-1}(Int(Y - V)) = \phi$ which implies that $f(U) \cap Int(Y - V) = \phi$. Then $f(U) \subseteq Y - Int(Y - V)$, so $f(U) \subseteq Cl(V)$, which means that f is weakly ωb-continuous. □

THEOREM 2.10. *Let $f : (X, \tau) \to (Y, \rho)$ be a surjection function such that $f(U)$ is $\omega b - open$ in Y for any $\omega b - open$ set U in X and let $g : (Y, \rho) \to (Z, \sigma)$ be any function. If gof is weakly ωb-continuous then g is weakly ωb-continuous.*

Proof. Let $y \in Y$. Since f is surjection, there exists $x \in X$ such that $f(x) = y$. Let $V \in \sigma$ with $g(y) \in V$, so $(gof)(x) \in V$. Since gof is weakly ωb-continuous there exists an $\omega b - open$ set U in X containing x such that $(gof)(U) \subseteq Cl(V)$. By assumption $H = f(U)$ is an $\omega b - open$ set in Y and contains $f(x) = y$. Thus $g(H) \subseteq Cl(V)$. Hence g is weakly ωb-continuous. □

DEFINITION 2.11. *A function $f : (X, \tau) \to (Y, \rho)$ is called ωb-irresolute if $f^{-1}(V)$ is $\omega b - open$ in (X, τ) for every $\omega b - open$ set V in (Y, ρ).*

THEOREM 2.12. *If $f : (X, \tau) \to (Y, \rho)$ is ωb-irresolute and $g : (Y, \rho) \to (Z, \sigma)$ is weakly ωb-continuous then $gof : (X, \tau) \to (Z, \sigma)$ is weakly ωb-continuous.*

Proof. Let $x \in X$ and $V \in \sigma$ such that $(gof)(x) = g(f(x)) \in V$. Let $y = f(x)$. Since g is weakly ωb-continuous. So there exists an $\omega b - open$ set W in Y such that $y \in W$ and $g(W) \subseteq Cl(V)$. Let $U = f^{-1}(W)$. Then U is an $\omega b - open$ set in X as f is ωb-irresolute. Now $(gof)(U) = g(f(f^{-1}(W))) \subseteq g(W)$. Then $x \in U$ and $(gof)(U) \subseteq Cl(V)$. Hence gof is weakly ωb-continuous. □

THEOREM 2.13. *If $f : (X, \tau) \to (Y, \rho)$ is weakly ωb-continuous and $g : (Y, \rho) \to (Z, \sigma)$ is continuous then $gof : (X, \tau) \to (Z, \sigma)$ is weakly ωb-continuous.*

Proof. Let $x \in X$ and W be an open set in Z containing $(gof)(x) = g(f(x))$. Then $g^{-1}(W)$ is an open set in Y containing $f(x)$. So there exists an $\omega b - open$ set U in X containing x such that $f(U) \subseteq Cl(g^{-1}(W))$. Since g is continuous we have $(gof)(U) \subseteq g(Cl(g^{-1}(W))) \subseteq g(g^{-1}(Cl(W))) \subseteq Cl(W)$. □

THEOREM 2.14. *A function $f : X \to Y$ is weakly ωb-continuous if and only if the graph function $g : X \to X \times Y$ of f defined by $g(x) = (x, f(x))$ for each $x \in X$, is weakly ωb-continuous.*

Proof.

$\Rightarrow$) Suppose that f is weakly ωb-continuous. Let $x \in X$ and W be an open set in $X \times Y$ containing $g(x)$. Then there exists a basic open set $U_1 \times V$ in $X \times Y$ such that $g(x) = (x, f(x)) \in U_1 \times V \subseteq W$. Since f is weakly ωb-continuous there exists an $\omega b - open$ set U_2 in X containing x such that $f(U_2) \subseteq Cl(V)$. Let $U = U_1 \cap U_2$ then U is an $\omega b - open$ set in X with $x \in U$ and $g(U) \subseteq Cl(W)$.

$\Leftarrow$) Suppose that g is weakly ωb-continuous. Let $x \in X$ and V be an open set in Y containing $f(x)$. Then $X \times V$ is an open set containing $g(x)$ and hence there exists an $\omega b - open$ set U in X containing x such that $g(U) \subseteq Cl(X \times V) = X \times Cl(V)$. Therefore, we have $f(U) \subseteq Cl(V)$ and hence f is weakly ωb-continuous. □

THEOREM 2.15. *If $f : (X, \tau) \to (Y, \rho)$ is a weakly ωb-continuous function and Y is Hausdorff then the set $G(f) = \{(x, f(x) : x \in X\}$ is an $\omega b - closed$ set in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$. Since Y is Hausdorff, there exist two disjoint open sets U and V such that $y \in U$ and $f(x) \in V$. Since f is weakly ωb-continuous, there exists an $\omega b - open$ set W containing x such that $f(W) \subseteq Cl(V)$. Since V and U are disjoint, we have $U \cap Cl(V) = \phi$ and hence $U \cap f(W) = \phi$. This shows that $(W \times U) \cap G(f) = \phi$. Then $G(f)$ is $\omega b - closed$. □

THEOREM 2.16. *If $f : X_1 \to Y$ is ωb-continuous, $g : X_2 \to Y$ is weakly ωb-continuous and Y is Hausdorff, then the set $A = \{(x_1, x_2) \in X_1 \times X_2 : f(x_1) = g(x_2)\}$ is $\omega b - closed$ in $X_1 \times X_2$.*

Proof. Let $(x_1, x_2) \in (X_1 \times X_2) - A$. Then $f(x_1) \neq g(x_2)$ and there exist open sets V_1 and V_2 in Y such that $f(x_1) \in V_1$, $g(x_2) \in V_2$ and $V_1 \cap V_2 = \phi$, hence $V_1 \cap Cl(V_2) = \phi$. Since f is ωb-continuous there exists an $\omega b - open$ set U_1 in X_1 containing x_1 such that $f(U_1) \subseteq V_1$. Since g is weakly ωb-continuous there exists an $\omega b - open$ set U_2 in X_2 containing x_2 such that $g(U_2) \subseteq Cl(V_2)$. Now $U_1 \times U_2$ is an $\omega b - open$ set in $X_1 \times X_2$ with $(x_1, x_2) \in U_1 \times U_2 \subseteq (X_1 \times X_2) - A$. This shows that A is $\omega b - closed$ in $X_1 \times X_2$. □

THEOREM 2.17. *If (Y, ρ) is a regular space then a function $f : (X, \tau) \to (Y, \rho)$ is weakly ωb-continuous if and only if it is ωb-continuous*

Proof.

⇒) Let x be any point in X and V be any open set in Y containing $f(x)$. Since (Y, ρ) is regular, there exists $W \in \rho$ such that $f(x) \in W \subseteq Cl(W) \subseteq V$. Since f is weakly ωb-continuous there exists an $\omega b - open$ set U in X containing x such that $f(U) \subseteq Cl(W)$. So $f(U) \subseteq V$. Therefore, f is ωb-continuous.

⇐) Clear. □

DEFINITION 2.18. *Any weakly ωb-continuous function $f : X \to A$, where $A \subseteq X$ and $f_A = f\mid_A$ is the identity function on A, is called weakly ωb-continuous retraction.*

THEOREM 2.19. *Let $f : X \to A$ be a weakly ωb-continuous retraction of X onto A where $A \subseteq X$. If X is a Hausdorff space, then A is an $\omega b - closed$ set in X.*

Proof. Suppoe that A is not $\omega b - closed$ in X. Then there exist a point $x \in \omega bCl(A) - A$. Since f is weakly ωb-continuous retraction, we have $f(x) \neq x$. Since X is Hausdorff, there exist two disjoint open sets U and V such that $x \in U$ and $f(x) \in V$. Then we have $U \cap Cl(V) = \phi$. Now, Let W be any $\omega b - open$ set in X containing x. Then $U \cap W$ is an $\omega b - open$ set containing x and hence $(U \cap W) \cap A \neq \phi$ because $x \in \omega bCl(A)$. Let $y \in (U \cap W) \cap A$. Since $y \in A$, $f(y) = y \in U$ and hence $f(y) \notin Cl(V)$. This gives that $f(W)$ is not a subset of $Cl(V)$. This contradicts the fact that f is weakly ωb-continuous. Therefore A is $\omega b - closed$ in X. □

DEFINITION 2.20. *A space X is called:*

(a) $\omega b - T_1$ if for each pair of distinct points x and y in X, there exist two $\omega b - open$ sets U and V of X containing x and y, respectively, such that $y \notin U$ and $x \notin V$.

(b) $\omega b - T_2$ if for each pair of distinct points x and y in X, there exist two $\omega b - open$ sets U and V of X containing x and y, respectively, such that $U \cap V = \phi$

Theorem 2.21. *If for each pair of distinct points x and y in a space X there exists a function f of X into a Hausdorff space Y such that*

1) $f(x) \neq f(y)$
2) f *is ωb-continuous at x and*
3) f *is weakly ωb-continuous at y,*

then X is $\omega b - T_2$.

Proof. Since $f(x) \neq f(y)$ and Y is Hausdorff, there exist open sets V_1 and V_2 of Y containing $f(x)$ and $f(y)$, respectively, such that $V_1 \cap V_2 = \phi$, hence $V_1 \cap Cl(V_2) = \phi$. Since f is ωb-continuous at x, there exists an $\omega b - open$ set U_1 in X containing x such that $f(U_1) \subseteq V_1$. Since f is weakly ωb-continuous at y, there exists an $\omega b - open$ set U_2 in X containing y such that $f(U_2) \subseteq Cl(V_2)$. Therefore we obtain $U_1 \cap U_2 = \phi$. This shows that X is $\omega b - T_2$. □

Definition 2.22. *A space X is called Urysohn [5] if for each pair of distinct points x and y in X, there exist open sets U and V such that $x \in U$, $y \in V$ and $Cl(U) \cap Cl(V) = \phi$.*

Theorem 2.23. *Let $f : (X, \tau) \to (Y, \rho)$ be a weakly ωb-continuous injection. Then the following hold:*

(a) *If Y is Hausdorff, then X is $\omega b - T_1$.*
(b) *If Y is Urysohn, then X is $\omega b - T_2$.*

Proof.

(a) Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Then $f(x_1) \neq f(x_2)$ and there exist open sets V_1 and V_2 in Y containing $f(x_1)$ and $f(x_2)$, respectively, such that $V_1 \cap V_2 = \phi$. Then we obtain $f(x_1) \notin Cl(V_2)$ and $f(x_2) \notin Cl(V_1)$. Since f is weakly ωb-continuous, there exist $\omega b - open$ sets U_1 and U_2 with $x_1 \in U_1$ and $x_2 \in U_2$ such that $f(U_1) \subseteq Cl(V_1)$ and $f(U_2) \subseteq Cl(V_2)$. Hence we obtain $x_2 \notin U_1$ and $x_1 \notin U_2$. This shows that X is $\omega b - T_1$.

(b) Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Then $f(x_1) \neq f(x_2)$ and there exist open sets V_1 and V_2 in Y containing $f(x_1)$ and $f(x_2)$, respectively, such that $Cl(V_1) \cap Cl(V_2) = \phi$. Since f is weakly ωb-continuous there exist $\omega b - open$ sets U_1 and U_2 in X with $x_1 \in U_1$ and $x_2 \in U_2$ such that $f(U_1) \subseteq Cl(V_1)$ and $f(U_2) \subseteq Cl(V_2)$.Since $f^{-1}(Cl(V_1)) \cap f^{-1}(Cl(V_2)) = \phi$ we obtain $U_1 \cap U_2 = \phi$. Hence X is $\omega b - T_2$. □

Definition 2.24. *A function $f : X \to Y$ is said to have a strongly $\omega b - closed$ graph if for each $(x, y) \in (X \times Y) - G(f)$ there exist an $\omega b - open$ subset U of X and an open subset V of Y such that $(x, y) \in U \times V$ and $(U \times Cl(V)) \cap G(f) = \phi$.*

THEOREM 2.25. *If Y is a Urysohn space and $f : X \to Y$ is weakly ωb-continuous, then $G(f)$ is strongly $\omega b - closed$.*

Proof. Let $(x, y) \in (X \times Y) - G(f)$. Then $y \neq f(x)$ and there exist open sets V and W in Y with $f(x) \in V$ and $y \in W$ such that $Cl(V) \cap Cl(W) = \phi$. Since f is weakly ωb-continuous, there exists an $\omega b - open$ subset U of X containing x such that $f(U) \subseteq Cl(V)$. Therefore we obtain $f(U) \cap Cl(W) = \phi$ and hence $(U \times Cl(W)) \cap G(f) = \phi$. This shows that $G(f)$ is strongly $\omega b - closed$ in $X \times Y$. □

THEOREM 2.26. *Let $f : (X, \tau) \to (Y, \rho)$ be a weakly ωb-continuous function having strongly $\omega b - closed$ graph $G(f)$. If f is injective, then X is $\omega b - T_2$.*

Proof. Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is injective, $f(x_1) \neq f(x_2)$ and $(x_1, f(x_2)) \notin G(f)$. Since $G(f)$ is strongly $\omega b - closed$, there exist an $\omega b - open$ subset U of X containing x_1 and an open subset V of Y such that $(x_1, f(x_2)) \in U \times V$ and $(U \times Cl(V)) \cap G(f) = \phi$ and hence $f(U) \cap Cl(V) = \phi$. Since f is weakly ωb-continuous, there exists an $\omega b - open$ subset W of X containing x_2 such that $f(W) \subseteq Cl(V)$. Therefore, we have $f(U) \cap f(W) = \phi$ and hence $U \cap W = \phi$. This shows that X is $\omega b - T_2$. □

DEFINITION 2.27. *A space X is said to be ωb-connected if X can not be written as a union of two non-empty disjoint $\omega b - open$ sets.*

THEOREM 2.28. *If X is an ωb-connected space and $f : X \to Y$ is weakly ωb-continuous surjection then Y is connected.*

Proof. Suppose that Y is not connected. Then there exist two non-empty disjoint open sets U and V in Y such that $U \cup V = Y$. Hence, we have $f^{-1}(U) \cap f^{-1}(V) = \phi$, $f^{-1}(U) \cup f^{-1}(V) = X$ and since f is surjection we have $f^{-1}(U) \neq \phi \neq f^{-1}(V)$. By Theorem 2.8, we have $f^{-1}(U) \subseteq \omega bInt[f^{-1}(Cl(U))]$ and $f^{-1}(V) \subseteq \omega bInt[f^{-1}(Cl(V))]$. Since U and V are clopen we have $f^{-1}(U) \subseteq \omega bInt[f^{-1}(U)]$ and $f^{-1}(V) \subseteq \omega bInt[f^{-1}(V)]$ and hence $f^{-1}(U)$ and $f^{-1}(V)$ are $\omega b - open$. This implies that X is not ωb-connected which is a contradiction. Therefore Y is connected. □

DEFINITION 2.29. *A topological space (X, τ) is said to be:*

(a) almost compact [3] if every open cover of X has a finite subfamily whose closures cover X.

(b) almost Lindelöf [6] if every open cover of X has a countable subfamily whose closures cover X.

DEFINITION 2.30. *A topological space (X, τ) is said to be ωb-compact (resp. ωb-Lindelöf) if every $\omega b - open$ cover of X has a finite (resp. countable) subcover.*

Theorem 2.31. *Let $f : X \to Y$ be a weakly ωb-continuous surjection. Then the following hold:*

(a) If X is ωb-compact, then Y is almost compact.

(b) If X is ωb-Lindelöf, then Y is almost Lindelöf.

Proof.

(a) Let $\{V_\alpha : \alpha \in \Delta\}$ be a cover of Y by open sets in Y. For each $x \in X$ there exists $V_{\alpha_x} \in \{V_\alpha : \alpha \in \Delta\}$ such that $f(x) \in V_{\alpha_x}$. Since f is weakly ωb-continuous, there exists an $\omega b - open$ set U_x of X containing x such that $f(U_x) \subseteq Cl(V_{\alpha_x})$. The family $\{U_x : x \in X\}$ is a cover of X by $\omega b - open$ sets of X and hence there exists a finite subset X_0 of X such that $X \subseteq \cup\{U_x : x \in X_0\}$. Therefore, we obtain $Y = f(X) \subseteq \cup\{Cl(V_{\alpha_x}) : x \in X_0\}$. This shows that Y is almost compact.

(b) Similar to *(a)*. □

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