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Cardinal Invariants for Function Spaces

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Summary. - *Several function space topologies can be generated by a procedure defined by two parameters: a network on the domain and a topology on the hyperspace of the range. Results about cardinal functions and metrizability for a particular class of such spaces are given.*

1. Introduction

Let X, Y be topological spaces, $CL(Y)$ the set of all closed nonempty subsets of Y and $C(X,Y)$ the set of all continuous functions from X to Y. Any network α in X and any hypertopology τ on $CL(Y)$ induce on $C(X,Y)$ a topology τ_α by requiring : "A net $\{f_\lambda\}$ in $C(X,Y)$ τ_α-converges to $f \in C(X,Y)$ iff the net $\{\overline{f_\lambda(A)}\}$ τ-converges to $\overline{f(A)}$ in $CL(Y)$ for all $A \in \alpha$". In other words τ_α has the following subbasic open sets:

$$[A : \mathcal{G}] = \{f \in C(X,Y) : \overline{f(A)} \in \mathcal{G}\}$$

where $A \in \alpha$ and $\mathcal{G}$ is open in τ.
This is a general method to produce function space topologies by using hypertopologies (see [8]).

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In particular, given the well known Vietoris topology τ_V on $CL(Y)$, any network α on X produces a topology on $C(X,Y)$, which we denote by $\tau_{\alpha,V}$. The function space having this topology is denoted by $C_{\alpha,V}(X,Y)$ or simply $C_{\alpha,V}(X)$ when $Y = \mathbb{R}$.
If α contains the singletons, a typical subbasic open set in $\tau_{\alpha,V}$ is

$$[A : V] = \{f \in C(X,Y) : \overline{f(A)} \subset V\}$$

where $A \in \alpha$ and V is open in Y (see [4]).

A topology on $C(X,Y)$ is called "set-open" provided there is some closed network α such that the family $\{[A,V] : A \in \alpha$ and V is open in $Y\}$, where $[A,V] = \{f \in C(X,Y) : f(A) \subset V\}$, is a subbase for the topology (see [7]).
This function space is usally denoted by $C_\alpha(X,Y)$ ($C_\alpha(X)$ when $Y = \mathbb{R}$).

The relations between $C_\alpha(X,Y)$ and $C_{\alpha,V}(X,Y)$ depend essentially on the choice of the network α. For example, if α is compact they coincide.
McCoy and Ntantu in [7] give results about relations between some cardinal functions on $C_\alpha(X)$ and the domain X when α is a compact hereditarily closed network and $Y = \mathbb{R}$.

Trying to prove such relations for $C_{\alpha,V}(X)$, we find results that have as corollaries not only those obtained by McCoy and Ntantu but also fundamental theorems about countability properties of the most famous set-open topologies, such as the uniform convergence topology on compact sets and the pointwise convergence topology (see [2]). In this case the function spaces having these topologies are denoted respectively by $C_K(X)$ and $C_p(X)$.
Furthermore simple necessary and sufficient conditions on metrizability of $C_{\alpha,V}(X,Y)$ are given. As corollaries we obtain the well known theorems about metrizability of $C_p(X)$ and $C_K(X)$ (see [1]).

Throughout this paper all spaces are assumed to be Hausdorff and all networks to contain singletons.

We refer the reader to [5] for notations and terminology not explicitly given.

2. Preliminaries

Let X, Y be topological spaces, $CL(Y)$ the set of all closed nonempty subsets of Y and $C(X, Y)$ the set of all continuous functions from X to Y. A nonempty family α of nonempty subsets of X is a network on X if for every $x \in X$ and for every open neighbourhood U of x there exists an $A \in \alpha$ such that $x \in A \subset U$.
It is called closed (compact) provided each member is closed (compact). A closed network is called hereditarily closed iff every closed subset of a member is a member.

A network α on X and a topology τ in $CL(Y)$ induce in $C(X, Y)$ a natural convergence which topologizes $C(X, Y)$ by requiring: $\{f_\lambda\}$ τ_α-converges to f in $C(X, Y)$ iff $\{\overline{f_\lambda(A)}\}$ τ-converges to $\overline{f(A)}$ in $CL(Y)$ for each member A in α (see [4] and [8]).
The method establishes the correspondence $(\alpha, \tau) \longrightarrow \tau_\alpha$. Clearly, given a topology τ on $CL(Y)$ the choice of the network α plays an important role to construct the associated function space topology. This role is emphasized by introducing the following definitions:

Definition 2.1. *A network α is Y-closed iff for every function $f \in C(X, Y)$ and $A \in \alpha$ $\quad \overline{f(A)} = f(A)$.*

For example a compact network on X is Y-closed when Y is a Hausdorff space. There exist Y-closed networks, with Y Hausdorff, that are not compact.

Definition 2.2. *A network α is Y-compact iff for every function $f \in C(X, Y)$ and $A \in \alpha$, $\overline{f(A)}$ is compact.*

If $X = Y = R$ the set of bounded intervals of X is a Y-compact network on X which is neither compact nor Y-closed.
Clearly, if Y is compact then every network on X is Y-compact.

Definition 2.3. *A network α on X is functionally separating iff for every $A \in \alpha$ and B nonempty closed in X such that $A \cap B = \emptyset$ there exists a continuous function $f : X \to [0, 1]$ such that $f(A) = \{0\}$ and $f(B) = \{1\}$.*

If X is completely regular (normal) then every compact (closed) network on X is functionally separating.

DEFINITION 2.4. *A network α on X is regular with respect to Y iff for every $A \in \alpha$, $f \in C(X,Y)$ and V open in Y such that $\overline{f(A)} \subset V$ there exists an open set U in Y such that $\overline{f(A)} \subset U \subset \overline{U} \subset V$.*

If Y is normal then every network on X is regular with respect to Y.

If τ is the Vietoris topology, these networks produce various function space topologies.
Each topology is denoted by $\tau_{\alpha,V}$ and the related function space by $C_{\alpha,V}(X,Y)$.
The Vietoris topology τ_V on $CL(Y)$ has as subbasic open sets

$$\begin{aligned} V^- &= \{E \in CL(Y) : E \cap V \neq \emptyset\} \\ W^+ &= \{E \in CL(Y) : E \subset W\} \end{aligned}$$

where V, W are open sets in Y.
We may think of τ_V as obtained by joining the topologies τ_{V^-} and τ_{V^+} generated taking as basic open sets the family $\{V^- : V \text{ open in } Y\}$ and $\{W^+ : W \text{ open in } Y\}$ respectively (see [3]).

If Y is metrizable then $CL(Y)$ can be metrized by Hausdorff metric. Recall that if d is a compatible metric on Y, for every $A, B \in CL(Y)$ the metric

$$d_H(A,B) = \max\{\sup_{a \in A} d(a,B), \sup_{b \in B} d(b,A)\}$$

is the Hausdorff metric on $CL(Y)$.
Recall also that if $\Im$ is a family of compact subsets of Y then τ_V coincides with the topology induced by Hausdorff metric on $\Im$ (see [3]).

If we endow the hyperspace of Y with τ_{V^+} we can costruct a topology on $C(X,Y)$ in the following way: for every A belonging to a network α and open set V in Y, define

$$[A : V] = \{f \in C(X,Y) : \overline{f(A)} \in V^+\}$$

equivalently

$$[A : V] = \{f \in C(X,Y) : \overline{f(A)} \subset V\}.$$

The family $\{[A:V] : A \in \alpha, V$ open in $Y\}$ is a subbase for a topology on $C(X,Y)$.
If Y is regular or α contains the singletons of X, then this topology is exactly $\tau_{\alpha,V}$ (see [4] Proposition 1.2).

For almost any natural topology on $C(X,Y)$ the topological properties of X and Y interact with the topological properties of $C(X,Y)$. So after comparing $C_{\alpha,V}(X,Y)$ with $C_\alpha(X,Y)$, introduced in [7], we deduce results about these interactions for $C_{\alpha,V}(X,Y)$.

3. Comparison between $\mathbf{C}_{\alpha,\mathbf{V}}(\mathbf{X},\mathbf{Y})$ and $\mathbf{C}_\alpha(\mathbf{X},\mathbf{Y})$

In [7] $C_\alpha(X,Y)$ is a topological space generated taking as subbasic open sets the following sets:

$$[A,V] = \{f \in C(X,Y) : f(A) \subset V\}$$

where $A \in \alpha$ and V is open in Y.
Clearly, if α is Y-closed then $C_{\alpha,V}(X,Y) = C_\alpha(X,Y)$. In particular if α is the network of all compact subsets of X and Y is Hausdorff these spaces coincide both with $C_k(X,Y)$. In general $C_\alpha(X,Y)$ is distinct from $C_{\alpha,V}(X,Y)$.

Proposition 3.1. *If α is a regular network on X then $C_{\alpha,V}(X,Y) \subset C_\alpha(X,Y)$.*

The next example shows that this is a strict inclusion when $Y = \mathbb{R}$ and $\alpha = CL(Y)$.

Example 3.2. *Let $X = Y = \mathbb{R}$ equipped with the natural topology and $\alpha = CL(Y)$. There exists a C_α-open set that is not open in $\tau_{\alpha,V}$.*

Consider the function $f : X \to Y$ defined by $f(x) = \operatorname{arctg} x$ for every $x \in X$. If $A = [\frac{\pi}{2}, +\infty[$ and $V =]-\frac{\pi}{2}, \frac{\pi}{2}[$, then $f \in [A,V]$, but every $\tau_{\alpha,V}$-open set containing f is not contained in $[A,V]$. Infact, let $A_1, \ldots, A_n$ be elements of α and $V_1, \ldots, V_n$ open sets in Y such that $f \in \bigcap_{i=1}^{n} [A_i : V_i]$.
We distinguish two cases: 1) $\frac{\pi}{2} \in \overline{f(\bigcup_{i=1}^{n} A_i)}$; 2) $\frac{\pi}{2} \notin \overline{f(\bigcup_{i=1}^{n} A_i)}$.

It can be shown that in both cases it results $(\bigcap_{i=1}^{n}[A_i : V_i])\backslash[A, V] \neq \emptyset$.

1) If $\frac{\pi}{2} \in \overline{f(\bigcup A_i)}$ there exists $j \in \{1, \ldots, n\}$ such that $\frac{\pi}{2} \in \overline{f(A_j)}$, so A_j has no upper bounds. Suppose that it is the only element of $\{A_i : i = 1, \ldots, n\}$ to have no upper bounds. Then there exists $x_0 \in [\frac{\pi}{2}, +\infty[$ such that $(\bigcup_{i \neq j} A_i) \cap [x_0, +\infty[= \emptyset$.

Let $p = \sup\{x \in \mathbb{R} : x \in \bigcup_{i \neq j} A_i\}$. The function

$$g(x) = \begin{cases} f(x) & x \in]-\infty, p] \\ \frac{\pi}{2} + \frac{\frac{\pi}{2} - f(p)}{x_0 - p}(x - x_0) & x \in [p, x_0] \\ f_{\frac{\pi}{2}} & x \in [x_0, +\infty[\end{cases}$$

where $f_{\frac{\pi}{2}}$ denotes the constant function related to $\frac{\pi}{2}$, belongs to $\cap[A_i : V_i]$ but it does not belong to $[A, V]$.

Observe that if A_j is not the only unbounded element of the set $\{A_1, \ldots, A_n\}$ we can take their intersection and procede as in the previous way. If all A_i's have no upper bound the function $f_{\frac{\pi}{2}}$ belongs to $\cap[A_i : V_i]$ but it does not belong to $[A, V]$.

2) If $\frac{\pi}{2} \notin \overline{f(\cup A_i)}$ then for every $i \in \{1, \ldots, n\}$ A_i is upper bounded so there exist $x_0 \in \mathbb{R}$ such that $x_0 > \max \cup_{i=1}^{n} A_i$. Let $g : (\cup A_i) \cup \{x_0\} \to Y$ be defined by

$$g(x) = f(x) \qquad x \in \cup A_i$$
$$g(x_0) = \frac{\pi}{2}$$

and let G be a continuous extension of g on X. Clearly $G \in \bigcap_{i=1}^{n}[A_i : V_i]\backslash[A, V]$.

4. Main results

Let X, Y be topological spaces and let α be a network on X. Before showing the interaction between cardinal functions on X and on $C_{\alpha,V}(X, Y)$ we need to recall the following definitions:

DEFINITION 4.1. [7] *An α-network β on X is a family of nonempty subsets of X such that for every $A \in \alpha$ and for every open neighbourhood U of A there exists a $B \in \beta$ such that $A \subset B \subset U$. An α-cover of a space X is a family of subsets of X such that every member of α is cointained in some member of this family.*

DEFINITION 4.2. [7] *The α-netweight, α-α-netweight and α-Arens number are respectively defined and denoted as follows: $\alpha nw(X) = \omega + \min\{|\mathcal{B}| : \mathcal{B}$ is an α-network for $X\}$, $\alpha\alpha nw(X) = \omega + \min\{|\mathcal{B}| : \mathcal{B} \subset \alpha$ and $\mathcal{B}$ is an α-network for $X\}$, $\alpha a(X) = \omega + \min\{|\mathcal{U}| : \mathcal{U} \subset \alpha$ and $\mathcal{U}$ is an α-cover for $X\}$.*

Recall that $K(Y)$ will denote the set of all nonempty compact subsets of Y.

LEMMA 4.3. *Let α be a network on X and let τ a topology on $CL(Y)$ then*

(1) $\quad w(C(X,Y),\tau_\alpha) \leq |\alpha| \cdot w(CL(Y),\tau)$;

(2) $\quad w(C(X,Y),\tau_\alpha) \leq |\alpha| \cdot w(K(Y),\tau)$ *if α is Y-compact.*

Proof. (1) Let $\mathcal{B}$ a base for $(C(L(Y),\tau)$ such that $|\mathcal{B}| = w(CL(Y),\tau)$. The family $\{[A : G] : A \in \alpha, G \in \mathcal{B}\}$ is a subbase for $(C(X,Y),\tau_\alpha)$ so $w(C(X,Y),\tau_\alpha) \leq |\alpha| \cdot |\mathcal{B}| = |\alpha| \cdot w(CL(Y),\tau)$.
In an analogous way it is easy to show the inequality (2). □

Note that in (2) the factor $|\alpha|$ cannot be omitted as the following example shows.

EXAMPLE 4.4. *Let $X = Y = \mathbb{R}, \alpha = \{\{x\} : x \in X\}$ and $\tau = \tau_V^+$. α is a Y-compact network on X and $w(K(Y),\tau_V^+) = \omega$. It suffices to observe that the family $\Im = \{V^+ : V$ open interval with rational bounds$\}$ is a base for $(K(Y),\tau_V^+)$. Moreover the family $\{[\{x\} : V] : x \in X,\ V \in \Im\}$ is a subbase for $\tau_{\alpha,V}$ so $w(C_{\alpha,V}(X,Y)) = c$.*

THEOREM 4.5. *Let α be a network on X and let $Y = \mathbb{R}$. Then*

(1) $\alpha\alpha nw(X) \leq w(C_{\alpha,V}(X,Y))$ *if α is functionally separating;*

(2) $\alpha\alpha nw(X) \geq w(C_{\alpha,V}(X,Y))$ *if α is Y-compact.*

Proof. First let us prove the inequality (1). Let $\mathcal{B}'$ a base of $C_{\alpha,V}(X,Y)$ of minimal cardinality: $|\mathcal{B}'| = w(C_{\alpha,V}(X,Y))$, contained in the fami- ly $\mathcal{B}$ of all finite intersections of members $[A : V]$ of the

canonical subbase S, and let S' be the family of all $[A:V]$ such that there is a $B' \in \mathcal{B}'$ of the form $B' = [A:V] \cap [A_1 : V_1] \cap \ldots \cap [A_k : V_k]$. Then it is easily seen that the family $\alpha' \subset \alpha$ of all A such that there is a V with $[A:V] \in S'$ is an α-network of cardinality $\leq w(C_{\alpha,V}(X,Y))$. In fact let $A \in \alpha, A \subset U, U$ open set.
Let $f \in C(X,Y)$ be such that $f(A) = \{0\}$ and $f(X \backslash U) = \{1\}$. We have $f \in [A :] - \frac{1}{4}, \frac{1}{4}[\ ])$, hence there is an $A' \in \alpha'$ such that $\overline{f} \in [A' : V] \subset [A :] - \frac{1}{4}, \frac{1}{4}[\ ]$.
Observe that $V \subset] - \frac{1}{4}, \frac{1}{4}[$. Clearly $A' \subset U$, since $f(X \backslash U) \cap V = \emptyset$. Finally $A \subset A'$. If not pick any $a \in A \backslash A'$. There exists a function g such that $g(A') = r \in V$ and $g(a) = 1$. To show the inequality (2) let $\alpha' \subset \alpha$ an α-network of minimal cardinality: $|\alpha'| = \alpha\alpha nw(X)$. It is easily seen that $\tau_{\alpha',V} = \tau_{\alpha,V}$. Fix $A \in \alpha, V$ open in Y and $f \in [A:V]$ and let U an open set in Y such that $\overline{f(A)} \subset U \subset \overline{U} \subset V$, then there is an $A' \in \alpha'$ such that $A \subset A' \subset f^{-1}(U)$ and $f \in [A' : V] \subset [A : V]$. Now it suffices to apply Lemma 4.3 (2) with $\tau = \tau_{V+}$ and recall that $w(K(Y), \tau_{V+}) = \omega : w(C_{\alpha,V}(X,Y)) = w(C_{\alpha',V}(X,Y)) \leq |\alpha'|$. □

Observe that: "If α is a Y-closed, Y-compact, functionally separating network on X and $Y = \mathbb{R}$ then $\alpha\alpha nw(X) = w(C_\alpha(X))$".
In particular, for compact networks on X, we deduce Theorem 4.5.2 in [7] without requiring the hereditarily closedness of α, that is to say:

COROLLARY 4.6. *Let α be a compact network on X. Then $\alpha\alpha nw(X) = w(C_\alpha(X))$.*

For Y-closed networks Theorem 4.5 is an effective generalization of Theorem 4.5.2 in [7] as the following example shows:

EXAMPLE 4.7. *A Y-closed, Y-compact, $Y = \mathbb{R}$, functionally separating network on a Tychonoff space X which is not compact.*

Let $Z = \prod_{t \in \mathbb{R}} Z_t$ where $Z_t = [0,1]\ \forall t \in \mathbb{R}$. The Σ-product $X = \Sigma(0) \subset Z$ is a countably compact normal space (see [5]),which is not locally compact.
Let α be a closed network on X.
- α is Y-closed and Y-compact: for every $A \in \alpha, A$ is countably compact then $f(A)$ is countably compact, that is to say compact.

- α is functionally separating because X is normal.
- α is not compact, otherwise X would be locally compact.

We need only to show that "For every compact network β $\tau_{\beta,V} \neq \tau_{\alpha,V}$".
(This follows from Theorem 1.1.1 [7], anyway we give a proof for the sake of completness.)

The network α is not compact, so let us take a non compact A in α. Let V be a nonempty proper subset of Y. Then $[A : V] = \{f \in C(X,Y) : \overline{f(A)} \subset V\}$ belongs to $\tau_{\alpha,V}$. Let $f \in [A : V]$, $K_1, \ldots, K_n \in \beta$ and $V_1, \ldots, V_n$ open in Y such that $f \in \bigcap_{i=1}^{n} [K_i : V_i]$. We show that $\cap[K_i : V_i]$ is not contained in $[A : V]$. A is not compact so there exists $a \in A$ such that $a \notin \bigcup_{i=1}^{n} K_i$. Fix $y \in Y \backslash V$ consider the function $g : (\bigcup_{i=1}^{n} K_i) \cup \{a\} \to Y$ defined by $g = f$ on $\bigcup_{i=1}^{n} K_i$ and $g(a) = y$. Let $\hat{g}$ be a continuous extension of g on X, clearly $\hat{g} \in (\bigcap_{i=1}^{n} [K_i : V_i]) \backslash [A : V]$.
Note that if α coincides with the set of all compact subsets of X, denoted by $K(X)$, then $C_{\alpha,V}(X,Y) = C_K(X,Y)$ and if $\alpha = \{\{x\} : x \in X\}$ then $C_{\alpha,V}(X,Y) = C_p(X,Y)$. In both cases α is functionally separating, hereditarily closed and applying Theorem 4.5 the second countability for $C_K(X)$ (respectively $C_p(X)$) is equivalent to the condition $\alpha\alpha nw(X) = \omega$ that is to say $\alpha a(X) = \omega$ and $\alpha nw(X) = \omega$ with $\alpha = K(X)$ (respectively $\alpha = \{\{x\} : x \in X\}$) by the relation $\alpha\alpha w(X) = \alpha a(X) \cdot \alpha nw(X)$ (Theorem 4.5.1 in [7]). Recall that when $\alpha = K(X)$ X is hemicompact iff $\alpha a(X) = \omega$, so we have the following:

Corollary 4.8. ([7] 4.5.3) *$C_K(X)$ is second countable iff X is hemicompact and $\alpha nw(X) = \omega$ with $\alpha = K(X)$.*

Corollary 4.9. ([7] 4.5.4; [2] I.3.7) *$C_p(X)$ is second countable iff X is countable.*

We consider now the metrizability of $C_{\alpha,V}(X,Y)$.

Theorem 4.10. *Let X be a Tychonoff space, Y a metrizable space*

containing an arc and let α be a closed, hereditarily closed, Y-compact network on X. Then the following conditions are equivalent:
(1) $C_{\alpha,V}(X,Y)$ is metrizable;
(2) $C_{\alpha,V}(X,Y)$ is first countable;
(3) $\alpha a(X) = \omega$.

Proof. (1)⇒(2). It is obvious.
(2)⇒(3). It follows from the inequality $\alpha a(X) \leq \chi(C_{\alpha,V}(X,Y))$ [7].
(3)⇒(1). Let ρ be a compatible metric on Y. If α is (Y,ρ)-compact then $\tau_{\alpha,V} = \tau_{\alpha,\rho}$ (see [4]), where $\tau_{\alpha,\rho}$ is the topology on $C(X,Y)$ generated by α and the Hausdorff metric topology induced by ρ_H on $CL(Y)$, and $\tau_{\alpha,\rho} = \tau_{\alpha,u.c.(\rho)}$ (see [4] and [8]), where $\tau_{\alpha,u.c.(\rho)}$ denotes the uniform convergence topology induced by ρ. On the other hand the hypothesis implies that there exist a countable family $\{A_n\}_{n\in Z^+}$ in α, such that for any A in α there is some $n \in Z^+$ such that $A \subset A_n$. Fix $n \in Z^+$ the metric

$$\rho_n(f,g) = \sup_{x\in A_n} \rho(f(x),g(x)) \qquad f,g \in C(X,Y)$$

induces the uniform convergence on A_n, and the following metric

$$\rho(f,g) = \sum_{n=1}^{\infty} \frac{1}{2^n}\rho_n(f,g) \quad f,g \in C(X,Y)$$

generates $\tau_{\alpha,u.c.}(\rho)$. □

Let X be a Tychonoff space and Y a metrizable space containing an arc.

COROLLARY 4.11. ([7] Exercise 9.1(a); [1]) *$C_k(X,Y)$ is metrizable iff X is hemicompact.*

COROLLARY 4.12. ([7] Exercise 9.1(b)) *$C_p(X)$ is metrizable iff X is countable.*

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