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Projection Constants of Almost-Milyutin Spaces

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Summary. - *We prove that there exist almost–Milyutin spaces whose projection constants are numbers of the form* $1+2\sum_{i=1}^{r}(1-\frac{1}{n_i})$, *where* $n_1,\ldots,n_r$ *are integers greater than* 1. *This generalizes our earlier results, where we showed the existence of almost–Milyutin spaces with exact projection constant greater or equal to* n, *for each positive integer* n.

1. Introduction

An almost–Milyutin space is a compact space T such that there exists an averaging operator for a continuous map from the generalized Cantor cube onto T (see more detailed definitions below). These spaces were introduced by Pełczyński in his monograph [3], in relation with the classification of continuous function spaces. The projection constant of T is the infimum of all norms of all averaging operators satisfying the definition. The constant is said to be exact if it is attained as the norm of some operator. Almost–Milyutin spaces with exact projection constant 1 are called Milyutin spaces, and they were previously studied by Milyutin [6]. Pełczyński showed the existence of almost–Milyutin spaces that are not Milyutin, but he was unable to compute projection constants. In particular he asked whether there are almost–Milutin spaces with projection constant greater or equal to n, for each positive integer n. This question was

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solved in [2] by using a theorem of Ditor [4] which provides (under certain hypotheses) a lower bound for the norm of every averaging operator for a given map. This bound has the form $1+2\sum_{i=1}^{r}(1-\frac{1}{n_i})$, where $n_1,\ldots,n_r$ are integers greater than 1. We gave certain conditions under which Ditor's theorem can be applied simultaneously to every continuous onto map from the generalized Cantor cube onto a space T. In the present paper we show that a refinement of our arguments allows us to construct almost–Milyutin spaces whose projection constants are any numbers of the form of those that appear in Ditor's theorem. These results are part of author's doctoral dissertation [5] prepared under the direction of Professor J.L. Blasco.

2. Preliminaries

Let S and T be compact spaces. Let $u : C(S) \longrightarrow C(T)$ be a (continuous) linear operator. Let $M(S)$ be the set of all regular finite (signed) Borel measures on S, which can be identified (by the Riesz representation theorem) with the topological dual space of $C(S)$. Namely, if $x \in C(S)^*$corresponds to the measure $\mu \in M(S)$, we have $x(f) = \int f d\mu$, for all $f \in C(S)$. We consider $M(S)$ endowed with the weak–star topology. We associate to the operator u a continuous map $\mu : T \longrightarrow M(S)$, given by $\mu_t = u^*(\delta_t)$, where $u^* : C(T)^* \longrightarrow C(S)^*$ is the dual operator determined by $u^*(x) = x \circ u$ and δ_t is the Dirac measure with support t.

The following proposition (see [3]) shows that linear operators are determined by their associated maps:

PROPOSITION 2.1. *Let S and T be compact spaces. Then*

a) For each linear operator $u : C(S) \longrightarrow C(T)$, the associated map $\mu : T \longrightarrow M(S)$ is continuous and for each $f \in C(S)$ and each $t \in T$ we have $u(f)(t) = \int f d\mu_t$.

b) If $\mu : T \longrightarrow M(S)$ is a continuous map, then $u : C(S) \longrightarrow C(T)$ defined by $u(f)(t) = \int f d\mu_t$ is a linear operator whose associated map is μ. Moreover, $\|u\| = \sup_{t\in T}\|\mu_t\|$, where the norm in $M(S)$ is given by $\|\mu\| = |\mu|(S)$.

An *averaging operator* for a continuous onto map $\phi : S \longrightarrow T$ is a linear operator $u : C(S) \longrightarrow C(T)$ such that $u(f \circ \phi) = f$, for each $f \in C(T)$.

The *generalized Cantor cube* is a space D^κ, where $D = \{0, 1\}$ is the discrete two–point space, κ is an infinite cardinal and D^κ has the product topology. A compact space T is an *almost–Milyutin space* if there exists a continuous map $\phi : D^\kappa \longrightarrow T$ from the generalized Cantor cube onto T for which there exists an averaging operator. The *projection constant* of an almost–Milyutin space T is the infimum $p(T)$ of all norms of all averaging operators for all continuous onto maps $\phi : D^\kappa \longrightarrow T$. When this infimum is attained by the norm of some operator, we say that the constant $p(T)$ is *exact.* If T is not an almost–Milyutin space we define $p(T) = +\infty$.

Let S be a compact space, let A be a directed set and let $\{C_\alpha\}_{\alpha\in A}$ be a net of subsets of S. We define $\limsup_\alpha C_\alpha = \bigcup_\beta \overline{\bigcap_{\alpha\geq\beta} C_\alpha}$. It is easy to prove that $\limsup_\alpha C_\alpha$ is the set of all cluster points of all nets $\{y_\alpha\}_{\alpha\in A}$ such that $y_\alpha \in C_\alpha$, for all index α. If S and T are compact spaces, $\phi : S \longrightarrow T$ is a continuous onto map and $\{t_\alpha\}_{\alpha\in A}$ is a net on T converging to $t \in T$, then $\limsup_\alpha \phi^{-1}(t_\alpha)$ is a nonempty compact subset of $\phi^{-1}(t)$.

Let S and T be compact spaces. Let $\phi : S \longrightarrow T$ be a continuous onto map. For each finite sequence $(n_1, \ldots, n_k)$ of integers greater than 1 (including the empty sequence of length 0) we define inductively the sets $M^\phi_{(n_1,\ldots,n_k)} \subset T$ by the following conditions:

1) $M^\phi_\emptyset = T$,

2) $M^\phi_{(n_1,\ldots,n_k)} = \{t \in T : \phi^{-1}(t)$ contains n_k disjoint sets of the form $\limsup_\alpha \phi^{-1}(t_\alpha)$, where $\{t_\alpha\} \subset M^\phi_{(n_1,\ldots,n_{k-1})}$ is a net converging to $t\}$.

The following theorem is due to Ditor [4], but this formulation is taken from Bade [1]:

THEOREM 2.2. *Let S and T be compact spaces and $\phi : S \longrightarrow T$ a continuous onto map. Let $(n_1, \ldots, n_k)$ be a finite sequence of integers greater than 1. If the set $M^\phi_{(n_1,\ldots,n_k)}$ is nonempty, then every* *averaging operator for ϕ has the norm greater than or equal to*

$$1+2\sum_{i=1}^{k}(1-\frac{1}{n_i}).$$

3. Lower bounds for projection constants

We recall that if X is a topological space and $A\subset X$, the G_δ–closure of A is the set $G_\delta(A)$ whose members are all points $x\in X$ such that every G_δ–subset of X containing x meets A.

DEFINITION 3.1. *Let T be a compact space. For each finite sequence $(n_1,\ldots,n_k)$ of integers greater than 1 (including the empty sequence of length 0) we define inductively the sets $M_{(n_1,\ldots,n_k)}\subset T$ by the following conditions:*

1) $M_\emptyset = T$,

2) $M_{(n_1,\ldots,n_k)}=\{p\in T$: there exist pairwise disjoint open subsets $U_1,\ldots,U_{n_k}$ of T such that $p\in\bigcap_{i=1}^{n_k}G_\delta(U_i\cap M_{(n_1,\ldots,n_{k-1})})\}$.

THEOREM 3.2. *Let T be a compact space and let $\phi : D^\kappa\longrightarrow T$ be a continuous map from the generalized Cantor cube onto T. If $(n_1,\ldots,n_k)$ is a finite sequence of integers greater than 1, then*

$$M_{(n_1,\ldots,n_k)}\subset M^{\phi}_{(n_1,\ldots,n_k)}.$$

Proof. By induction on k. For $k=0$ it is obvious. Now supose that $M_{(n_1,\ldots,n_k)}\subset M^{\phi}_{(n_1,\ldots,n_{k-1})}$ and take $p\in M_{(n_1,\ldots,n_k)}$. By definition, there exist pairwise disjoint open subsets $U_1,\ldots,U_{n_k}$ in T such that $p\in\bigcap_{i=1}^{n_k}G_\delta(U_i\cap M_{(n_1,\ldots,n_{k-1})})$. For each index i, consider the set

$$G_i=\overline{\phi^{-1}(U_i)}\cup\bigcup_{j\neq i}\overline{\phi^{-1}(U_j)}.$$

Since the closure of an open subset of D^κ depends on a countable set of coordinates (see [7]), it follows that these sets are compact G_δ sets. So there exist n_k decreasing families $\{W_i^k\}_{k=1}^{\infty}$ of clopen subsets of D^κ such that $G_1=\bigcap_{k=1}^{\infty}W_k^i$. Let us see that the following sentence is contradictory:

(*) *For each positive integer k, there exists an open neighborhood V_k^i of p such that for all $y \in V_k^i \cap U_i \cap M_{n_1,\dots,n_{k-1}}$, the set $\phi^{-1}(y) \cap W_k^i$ is nonempty.*

Asuming (*), the set $V^i = \bigcap_{k=1}^{\infty} V_k^i$ is a G_δ–subset of D^κ which contains p. Since p belongs to the G_δ–closure of $U_i \cap M_{n_1,\dots,n_{k-1}}$, there exists a point $y \in V^i \cap U_i \cap M_{n_1,\dots,n_{k-1}}$. By (*), for each positive integer k there exists a point $x_k^i \in \phi^{-1}(y) \cap W_k^i$. Take a cluster point z^i of the sequence$\{x_k^i\}_{k=1}^{\infty}$. Clearly, $z^i \in \phi^{-1}(y^i) \cap G$, and so $\phi(z^i) = y^i \in U_i$. However, on the other hand we have $\phi(z^i) \in \phi(G_i) \subset \bigcup_{j \neq i} \overline{U_j}$. And since the open sets U_i are pairwise disjoint, this is a contradiction.

Thus we see that (*) is false, i.e., for each index i there exists a positive integer k^i such that for every open neighborhood V of p there exists a point $y_V^i \in V \cap U_i \cap M_{n_1,\dots,n_{k-1}}$, satisfying that $\phi^{-1}(y_V^i) \cap W_k^i \neq \emptyset$. The nets $\{y_V^i\}_V$ converge to p and each set $L_i = \limsup_V \phi^{-1}(y_V^i)$ is contained in $(D^\kappa \setminus W_{k^i}^i) \cap \overline{\phi^{-1}(U_i)}$. Moreover, $G_i = \overline{\phi^{-1}(U_i)} \cap \bigcup_{j \neq i} \overline{\phi^{-1}(U_j)} \subset W_{k^i}^i$, it holds that L_i is disjoint with each $\overline{\phi^{-1}(U_j)}$, for $j \neq i$, and in particular the sets L_i are pairwise disjoint. This implies that p belongs to $M_{(n_1,\dots,n_k)}^{\phi}$ and the proof is complete. □

COROLLARY 3.3. *Let T be a compact space and $(n_1, \dots, n_k)$ a finite sequence of positive integers greater than 1. If the set $M_{(n_1,\dots,n_k)}$ is nonempty, then $p(T) \geq 1 + 2\sum_{i=1}^{k}(1 - \frac{1}{n_i})$.*

4. Construction of almost–Milyutin spaces

We need some lemmas. The first one is very easy and its proof is left to the reader.

LEMMA 4.1. *Let K be a compact space and S a clopen subset of K. Let $(n_1, \dots, n_r)$ be a finite sequence of integers greater than 1. Let $M_{(n_1,\dots,n_r)}^S$ and $M_{(n_1,\dots,n_r)}^K$ be the sets defined in the previous section for the spaces S and K, respectively. Then $M_{(n_1,\dots,n_r)}^S = M_{(n_1,\dots,n_r)}^K \cap S$.*

Lemma 4.2. *Let S_1, S_2, T_1 and T_2 be compact spaces and for $i = 1, 2$ let $\phi_i : S_i \longrightarrow T_i$ be a continuous onto map and $u_i : C(S_i) \longrightarrow C(T_i)$ an averaging operator for ϕ_i. Consider the map $\phi : S_1 \times S_2 \longrightarrow T_1 \times T_2$ given by $\phi(u, v) = (\phi_1(u), \phi_2(v))$. Then ϕ has an averaging operator u such that $\|u\| = \|u_1\| \, \|u_2\|$.*

Proof. Consider the tensor product $C(S_1) \bigotimes C(S_2)$, i.e., the subalgebra of $C(S_1 \times S_2)$ generated by the functions $f \otimes g : S_1 \times S_2 \longrightarrow \mathrm{R}$, given by $(f \otimes g)(u, v) = f(u)g(v)$. By the Stone–Weierstrass theorem, $C(S_1) \bigotimes C(S_2)$ is dense in $C(S_1 \times S_2)$. Consider also the maps $\mu_i : T_i \longrightarrow C(S_i)$ associated to the operators u_i. For each pair $t = (t_1, t_2) \in T_1 \times T_2$ we have the product measure $\mu_t = \mu_1(t_1) \otimes \mu_2(t_2) \in M(S_1 \times S_2)$, and so we have a continuous map $\mu : T_1 \times T_2 \longrightarrow M(S_1 \times S_2)$, from which we obtain a linear operator $u_0 : C(S_1 \times S_2) \longrightarrow B(T_1 \times T_2)$ (where $B(T_1 \times T_2)$ is the space of real–valued bounded functions on $T_1 \times T_2$) defined by $u_0(f)(t) = \int f d\mu_t$. It is easily seen that for each function of the form $f \otimes g \in C(S_1) \bigotimes C(S_2)$ we have $u_0(f \otimes g)(t_1, t_2) = u_1(f) \otimes u_2(g)$. So, if we call $u_1 \otimes u_2$ the restriction of u_0 to the tensor product $C(S_1) \bigotimes C(S_2)$ we have an operator $u_1 \otimes u_2 : C(S_1) \bigotimes C(S_2) \longrightarrow C(T_1) \bigotimes C(T_2)$. On the other hand, the integral representation of u_0 gives

$$\|u_0\| \leq \sup_{t \in T_1 \times T_2} \|\mu_t\| \leq \sup_{t_1 \in T_1} \|\mu_{t_1}\| \sup_{t_2 \in T_2} \|\mu_{t_2}\| = \|u_1\| \, \|u_2\|,$$

and so we have $\|u_1 \otimes u_2\| \leq \|u_1\| \, \|u_2\|$. The other inequality is clear.

Since every continuous linear operator is uniformly continuous, $C(T_1 \times T_2)$ is a complete space and $C(S_1) \bigotimes C(S_2)$ is dense in $C(S)$, we have that $u_1 \otimes u_2$ extends to a unique linear continuous operator $u : C(S) \longrightarrow C(T)$ such that $\|u\| = \|u_1\| \|u_2\|$.

Finally, we see that u is an averaging operator for ϕ. We must show that $u(f \circ \phi) = f$ for all $f \in C(T)$. Since $C(T_1) \bigotimes C(T_2)$ is dense in $C(T)$ it suffices to prove it for all $f \in C(T_1) \bigotimes C(T_2)$ and by linearity it suffices to prove it for all functions of the form $f \otimes g$, with $f \in C(T_1)$ and $g \in C(T_2)$. However, clearly $u(f \otimes g \circ \phi) = u((f \circ \phi_1) \otimes (g \circ \phi_2)) = u_1(f \circ \phi_1) \otimes u_2(g \circ \phi_2) = f \otimes g$. The Lemma is proved. □

Lemma 4.3. *Let $\{n_r\}_{r=1}^{\infty}$ be a sequence of integers greater than 1 and κ an uncountable cardinal. Then for each r, there exist a zero–*

dimensional compact space T in which no one–point subset is a G_δ–set, a point $p \in M_{(n_1,\dots,n_r)}$, a continuous onto map $\phi : D^\kappa \longrightarrow T$, and an averaging operator $u : C(D^\kappa) \longrightarrow C(T)$ for ϕ of norm $\lambda = 1 + 2\sum_{i=1}^r (1 - \frac{1}{n_i})$ and such that $|u(f)(p)| \leq \|f\|$ for all $f \in C(D^\kappa)$.

Proof. By induction on r. For $r = 0$ the lemma is satisfied taking $T = D^\kappa$, ϕ the identity map, p any point in T and u the identity operator. Asume we have constructed T, p, ϕ and u for a given r and let us see that there exist T', p', ϕ' and u' satisfying the hipotheses for $r + 1$.

Let $\psi : D^\kappa \times D^{\aleph_1} \longrightarrow T \times D^{\aleph_1}$ the continuous onto map given by $\psi(x, y) = (\phi(x), y)$. Since the identity operator is clearly an averaging operator for the identity map in $D^{\aleph_1}$, the previous lemma gives us an averaging operator $v : C(D^\kappa \times D^{\aleph_1}) \longrightarrow (T \times D^{\aleph_1})$ such that $\|v\| = \|u\|$ and $v(f \otimes g) = u(f) \otimes g$.

We shall see that $\{p\} \times D^{\aleph_1} \subset M_{(n_1,\dots,n_r)}$. In fact, we shall prove by (a second) induction on r that if $q \in M^T_{(n_1,\dots,n_r)}$ then $\{q\} \times D^{\aleph_1} \subset M^{T\times D^{\aleph_1}}_{(n_1,\dots,n_r)}$.

For $r = 0$ this is clear. If $q \in M^T_{(n_1,\dots,n_r)}$ then there exist pairwise disjoint open subsets $U_1, \dots, U_r$ such that $q \in \bigcap_{i=1}^r G_\delta(U_i \cap M^T_{(n_1,\dots,n_{r-1})})$. The sets $U_i \times D^{\aleph_1}$ are pairwise disjoint open subsets of $T \times D^{\aleph_1}$ and we are going to prove that for all $x \in D^{\aleph_1}$ we have $(q, x) \in \bigcap_{i=1}^r G_\delta(U_i \times D^{\aleph_1}) \cap M^{T\times D^{\aleph_1}}_{(n_1,\dots,n_{r-1})}$. Take a G_δ subset V of $T \times D^{\aleph_1}$ such that $(q, x) \in V$. Then $V = \bigcap_{n=1}^\infty V_n$ for certain open subsets V_n of $T \times D^{\aleph_1}$. For each n there exist open subsets A_n and B_n in T and $D^{\aleph_1}$, respectively, such that $(q, x) \in A_n \times B_n \subset V_n$. Thus $\bigcap_{n=1}^\infty A_n$ is a G_δ subset of T which contains q, and so $(\bigcap_{n=1}^\infty A_n) \cap U_i \cap M^T_{(n_1,\dots,n_{r-1})} \neq \emptyset$. Let $t \in (\bigcap_{n=1}^\infty A_n) \cap U_i \cap M^T_{(n_1,\dots,n_{r-1})}$. By the inductive hypothesis $(t, x) \in M^{T\times D^{\aleph_1}}_{(n_1,\dots,n_{r-1})}$, and so $(t, x) \in V \cap (U_1 \times D^{\aleph_1}) \cap M^{T\times D^{\aleph_1}}_{(n_1,\dots,n_{r-1})} \neq \emptyset$. This proves that $(q, x) \in \bigcap_{i=1}^r G_\delta(U_i \times D^{\aleph_1}) \cap M^{T\times D^{\aleph_1}}_{(n_1,\dots,n_{r-1})}$ and hence that$(q, x) \in M^{T\times D^{\aleph_1}}_{(n_1,\dots,n_r)}$.

Fix a point $x_0 \in D^{\aleph_1}$. Since $\{x_0\}$ is not a G_δ set in$D^{\aleph_1}$ we have $x_0 \in G_\delta(D^{\aleph_1} \setminus \{x_0\})$, and since we have seen that $\{p\} \times D^{\aleph_1} \subset M^{T\times D^{\aleph_1}}_{(n_1,\dots,n_r)}$, it is easy to check that $(p, x_0) \in G_\delta(((T \times D^{\aleph_1}) \setminus$

$\{(p,x_0)\}) \cap M^{T\times D^{\aleph_1}}_{(n_1,\ldots,n_r)})$.

On the other hand, if $f\otimes g \in C(T\times D^{\aleph_1})$ it holds that $|v(f\otimes g)(p,x_0)| = |u(f)(p)||g(x_0)| \leq \|f\|\|g\| = \|f\otimes g\|$. Using the density of $C(T)\bigotimes C(D^{\aleph_1})$ in $C(T\times D^{\aleph_1})$ and the continuity of v we obtain that $|v(h)(p,x_0)| \leq \|h\|$ for all $h\in C(T\times D^{\aleph_1})$.

Let $S_1,\ldots,S_{n_{r+1}}$ be disjoint copies of the space $T\times D^{\aleph_1}$. Let p_i be the point corresponding to (p,x_0) in each copy. So $p_i \in G_\delta((S_i\setminus\{p_i\})\cap M_{(n_1,\ldots,n_r)})$. Let $X_1,\ldots,X_{n_{r+1}}$ be disjoint copies of the space $D^\kappa\times D^{\aleph_1}$ and let $\psi_1 : X_i \longrightarrow S_i$ be the map corresponding to ψ. We have also averaging operators $v_i : C(X_i)\longrightarrow C(S_i)$ for the maps ψ_i, with the property that $|v_i(f)(p_i)| \leq \|f\|$ for all $f\in C(X_i)$.

Let T' be the space obtained by identifying the points p_i to a single point p' in the topological sum of the spaces S_i. Let X be the topological sum of the spaces X_i and let $\phi' : X' \longrightarrow T'$ be the map that restricted to each X_i coincides with ϕ_i. Note that T' is a compact zero–dimensional space in which no point is a G_δ set. If $x\neq p'$ is a point in some of the spaces S_i we can find a clopen subset U of S_i which contains x but not p'. By Lemma 4.1 we have that x belongs to the set $M_{(n_1,\ldots,n_r)}$ of S_i if and only if it belongs to the corresponding set of T'. Since $p'\in G_\delta((S_i\setminus\{p'\})\cap M_{(n_1,\ldots,n_r)})$ (where the G_δ–closure and the set M are taken in S_i), this is also true if we take the G_δ–closure and the set M in T'. So $p'\in\bigcap_{i=1}^{n_r+1} G_\delta((S_i\setminus\{p'\})\cap M_{(n_1,\ldots,n_r)})$ and the sets $S_i\setminus\{p'\}$ are pairwise open subsets of T'. Hence $p'\in M_{(n_1,\ldots,n_{r+1})}$.

We now define the operator $u' : C(X)\longrightarrow C(T')$ by $u'(f)|_{S_j} = v_j(f|_{X_j}) + \sum_{i=1}^{n_r+1}(\frac{1}{n_{r+1}} - \delta_{ij})v_i(f|_{X_i})(p')$,for each $j = 1,\ldots,n_{r+1}$ and each $f\in C(X)$. This is consistent because $u'(f)(p')$ is independent of the index j we use to calculate it. In fact,$u'(f)(p') = \sum_{i=1}^{n_r+1}\frac{1}{n_{r+1}}v_i(f|_{X_i})(p')$. It is easy to check that u' is indeed a linear continuous operator. By using the bounds $|v_i(f|_{X_i})(p')| \leq \|f\|$ we obtain that $\|u'\| \leq 1 + 2\sum_{i=1}^{r+1}(1-\frac{1}{n_1})$. From the expression for $u'(f)(p')$ we also obtain that $|u'(f)(p')| \leq \|f\|$, for all $f\in C(T')$.

We finally see that u' is an averaging operator for ϕ'. If we take $g\in C(T')$ we have $u'(g\circ\phi')|_{S_j}) v_j(g|_{S_j}\circ\psi_j) + \sum_{i=1}^{n_{r+1}}(\frac{1}{n_{r+1}} - \delta_{ij})v_i(g|_{S_i}\circ\psi_i)(p') = g|_{S_j} + \sum_{i=1}^{n_{r+1}}(\frac{1}{n_{r+1}} - \delta_{ij})g(p') = g|_{S_j} + 0\cdot g(p') = g|_{S_j}$, and hence $u'(g\circ\phi') = g$.

Now, since $M_{(n_1,\dots,n_{r+1})} \neq \emptyset$, we have that $\|u'\|$ is exactly $1 + 2\sum_{i=1}^{r+1}(1 - \frac{1}{n_i})$ and, since X is clearly homeomorphic to D^κ, lemma is proved. □

Our main result is a direct consequence of Lemma 4.3:

THEOREM 4.4. *Let $(n_1, \dots, n_r)$ be a finite sequence of integers greater than* 1*. Then there exists a zero–dimensional almost–Milyutin space T with exact projection constant $p(T) = 1 + 2\sum_{i=1}^{r}(1 - \frac{1}{n_i})$.*

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