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The π-weights and π-characters of Hyperspaces with the Hit-and-miss Topologies

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Summary. - *We prove that* $\pi w(\Delta(X)) = \pi\chi(\Delta(X)) = \max\{\pi w(X), \tau k(\Delta)\}$ *for the hit-and-miss topologies* τ_Δ *on the closed subsets of either a quasi-regular and* $\mathbf{R}_0$ *or a* T_1 *space* (X, τ)*, where* $\tau k(\Delta)$ *is a cardinal invariant associted with* Δ*.*

1. Introduction

No separation axioms for topological spaces are prior assumed, so later on we will always state them explicitly.

The hit-and-miss topologies on the non-empty closed subsets of a topological space X were first studied by Poppe in [14, 15], later were developed in the hands of Beer, Tamaki and Zsilinszky in [2, 3, 9]. To describe it, we introduce the following notation and terminology.

Let us use 2^X to denote the closed subsets of X and $\mathbf{K}(X)$ to denote the non-empty compact subsets of X. Let $CL(X) = 2^X \setminus \{\emptyset\}$ and $\mathcal{C}(X) = \{A \in \mathbf{K}(X) : A \text{ is closed }\}$. For a set S, we use $|S|$ to denote the cardinality of S. For a subset A of X, we denote the complement of A by A^c and define

$$A^- = \{F \in 2^X : F \cap A \neq \emptyset\} \text{ and } A^+ = \{F \in 2^X : F \cap A^c = \emptyset\}.$$

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Throughout this paper Δ always means a subset of $CL(X)$ satisfying the following properties:

a) Δ is closed with respect to the finite unions;

b) the intersection of every finite subset of Δ belongs to Δ if it contains an element of Δ.

Evidently, $CL(X)$ and $\mathcal{C}(X)$ satisfy both a) and b) respectively.

Let (X,τ) be a topological space, and let $\Delta \subseteq CL(X)$. The *hit-and-miss topology* τ_Δ on 2^X has as a subbase all sets of the form V^-, where $V \in \tau$, and of the form $(A^c)^+$, where $A \in \Delta$. For simplicity, denote by $\Delta(X)$ the topological space $(2^X, \tau_\Delta)$.

The topology generalizes the two most fundemental topologies on the closed subsets of a space, i.e., the Vietoris and Fell topogogies. In fact, if $\Delta = CL(X)$, then τ_Δ is the *Vietoris topology* τ_V on 2^X (cf. [10]); if $\Delta = \mathcal{C}(X)$, then τ_Δ is the *Fell topology* (cf. [6]) τ_F on 2^X.

Let (X,τ) be a topological space. A family $\gamma \subseteq \tau \setminus \{\emptyset\}$ is said to be a π-*basis* for X if each $U \in \tau \setminus \{\emptyset\}$ contains some $V \in \gamma$, and the π-*weight* $\pi w(X)$ is $\min\{|\gamma| : \gamma \text{ is a } \pi - \text{ basis for } X\} + \omega$. Let $p \in X$. A family $\gamma \subseteq \tau \setminus \{\emptyset\}$ is a *local* π-*basis for* X *at* p, if every open neighbourhood G of p contains some $V \in \gamma$; the π-*character of* X *at* p is defind as

$$\pi\chi(p,X) = \min\{|\mathcal{V}| : \mathcal{V} \text{ is a local } \pi \text{ basis for } X \text{ at } p\} + \omega.$$

The π-*character* of X is defined as follows:

$$\pi\chi(X) = \sup\{\pi\chi(p,X) : p \in X\} + \omega.$$

The purpose of this paper is to investigate the π-weight and π-character of $\Delta(X)$. We obtain a remarkable result that $\pi w(\Delta(X)) = \pi\chi(\Delta(X)) = \max\{\pi w(X), \tau k(\Delta)\}$ for either a quasi-regular $\mathbf{R}_0$ or a T_1 space X, where $\tau k(\Delta)$ is a cardinal invariant associated with Δ. As a result we get, for instance, that $\pi w(2^Q, \tau_F) = \pi\chi(2^Q, \tau_F) > \aleph_0$ where Q are the rational numbers (see, Corollary 3.4). In particular, this implies that $w(2^Q, \tau_F) > \aleph_0$ which is a known result. As another consequence of this our result, we get immeditely also that for a T_1 space X, $\pi w(X) \leq \pi w(\mathbf{K}(X), \tau_V) \leq \pi w((2^X, \tau_V)) = \pi w(X)$, i.e. $\pi w((\mathbf{K}(X), \tau_V)) = \pi w(X)$ which is a result of [11].

2. A cardinal invariant associated with Δ

The following notion is a key item throughout this paper and, it seems to be new.

DEFINITION 2.1. *Let (X,τ) be a topological space. For a subset $\mathcal{A}$ of $CL(X)$, we say $\mathcal{B} \subseteq \mathcal{A}$ is a τ-cofinal subset of $\mathcal{A}$ if, for any $W \in \tau$ and $A \in \mathcal{A}$ with $W \cap A^c \neq \emptyset$, there exists $B \in \mathcal{B}$ which contains A such that W hits B^c. The τ-cofinality $\tau k(\mathcal{A})$ of $\mathcal{A}$ is defined as follows:*

$$\tau k(\mathcal{A}) = \inf\{|\mathcal{B}| : \mathcal{B} \textit{ is a } \tau - \textit{ cofinal subset of } \mathcal{A}\} + \omega.$$

To justify this notion, we shall prove few facts about particular Δ. First of all, the $\tau-$cofinality of $\mathcal{A}$ is very similar to the cofinality $c(\mathcal{A})$ of $\mathcal{A}$. Let us recall that $c(\mathcal{A}) = \min\{|\mathcal{B}| : \mathcal{B}$ is a cofinal subset of $\mathcal{A}$ with respect to the "inclusion"$\} + \omega$.

PROPOSITION 2.2. *$c(\mathcal{A}) \leq \tau k(\mathcal{A})$ for each subset $\mathcal{A}$ of $CL(X)$.*

We omit the proof. Instead, we provide a counterexample showing that the inequality in Proposition 2.2 cannot be improved to be equal in general.

Let X be a set of cardinality $\aleph_1$ and let us endow it with the "cofinite" topology with respect to a fixed point $p \in X$. Namely, all points except p are isolated and $V \ni p$ is open if and only if the completment of V is finite. Now, let $\mathcal{A}$ be the compact subsets of X. Then the cofinality of $\mathcal{A}$ is $\aleph_0$, but the τ-cofinality of $\mathcal{A}$ is greater than $\aleph_0$. If a subset $\mathcal{B}$ of $\mathcal{A}$ is τ-cofinal, then $\mathcal{B}$ must contain the set $\{X \setminus \{x\} : x \neq p\}$ whose cardinality is equal to $\aleph_1$. Indeed, $\{x\}$ is open in X. If $A = X \setminus \{x\}$, then $A \in \mathcal{A}$ and $x \in A^c$. Since $\mathcal{B}$ is τ-cofinal, there is $B \in \mathcal{B}$ such that $x \in B^c$ and $B \supseteq X \setminus \{x\}$. Therefore, $X \setminus \{x\} = B \in \mathcal{B}$.

PROPOSITION 2.3. *Let (X,τ) be a non-trivial space. Let $\gamma \subseteq \tau \setminus \{\emptyset\}$. Then γ is a $\pi-$basis of X if and only if γ^c is $\tau-$ cofinal in $CL(X)$, where $\gamma^c = \{X \setminus G : G \in \gamma$ and $G \neq X\}$.*

Proof. "only if". Take $V \in \tau \setminus \{\emptyset\}$ and $A \in CL(X)$ such that $V \cap A^c \neq \emptyset$. As X is non-trivial, we can assume that $V \cap A^c \neq X$.

Since γ is a $\pi-$basis of X, there is $G \in \gamma$ such that $G \subseteq V \cap A^c$. If $B = G^c$, then $B \in \gamma^c$ and satisfies that $A \subseteq B$ and $V \cap B^c \neq \emptyset$.

"if". Let $U \in \tau \setminus \{\emptyset\}$. The case $U = X$ is trivial. Therefore, we assume that $U \neq X$. Thus $U^c \in CL(X)$. Now observe this pair (X, U^c). Since γ^c is $\tau-$ cofinal in $CL(X)$, there is $G \in \gamma$ such that $G^c \supseteq U^c$, i.e., $G \subseteq U$. □

By Proposition 2.3, we get immediately the following result:

Corollary 2.4. *Let (X, τ) be a topological space. Then*

$$\tau k(CL(X)) = \pi w(X).$$

To describe the next result, we recall the following notion introduced by Ntantu in [12].

The $k-k-$netweight $kknw(X)$ of X is defined as follows:

$$kknw(X) = \inf\{|\mathcal{A}| : \mathcal{A} \text{ is a } k\text{-network of } X \text{with compact closed members}\}.$$

Further, recall that a topological space is said to be *pseudo*-$\mathbf{R}_0$ (resp. $\mathbf{R}_0$) whenever each non-empty open subset of X contains the closure of some (resp. each) of its points (cf. [5]).

Proposition 2.5. *Let (X, τ) be a pseudo-$\mathbf{R}_0$ space. Then*

$$\tau k(\mathcal{C}(X)) \leq kknw(X).$$

Proof. Let $kknw(X) = \kappa$. We show that $\tau k(\mathcal{C}(X)) \leq \kappa$. Let $\mathcal{A} = \{K_\alpha : \alpha \in \kappa\} \subseteq \mathcal{C}(X)$ be a $k-k$-network of X. Take $K \in \mathcal{C}(X)$ and $W \in \tau \setminus \{\emptyset\}$ such that W hits K^c. As X is pseudo-$\mathbf{R}_0$, it follows that $\overline{\{x\}} \subseteq W \cap K^c$ for some $x \in W \cap K^c$. Hence, $K \subseteq X \setminus \overline{\{x\}}$. Since $\mathcal{A}$ is a $k-k$-network of X, there is $\alpha \in \kappa$ such that $K \subseteq K_\alpha \subseteq X \setminus \overline{\{x\}}$. Obviously, W hits K_α^c. □

The following example shows that the inequality in Proposition 2.5 cannot be improved to be equal in general.

Example 2.6. *Let $X = \beta\omega$. Accoding to Example 7.22 of [7], it follows that $\pi w(X) = \omega$ and $nw(X) > \omega$. Evidently, $nw(X) \leq kknw(X)$. Hence, $kknw(X) \geq 2^\omega$. On the other hand, by Corollary 2.4, we have $\tau k(\mathcal{C}(X)) \leq \pi w(X) = \omega$, because X is compact Hausdorff.*

3. Main results

Recall that a topological space is said to be *quasi-regular* (cf. [12]) whenever each non-empty open subset contains a closed subset whose interior is non-empty.

THEOREM 3.1. *Let (X,τ) be either a quasi-regular and $\mathbf{R}_0$ or a T_1 space. Then*

$$\pi w(\Delta(X)) = \pi\chi(\Delta(X)) = \max\{\pi w(X), \tau k(\Delta)\}.$$

For $\Delta = CL(X)$, Theorem 3.1 together with Corollary 2.4 imply the folloing result:

THEOREM 3.2. *Let X be either a quasi-regular and $\mathbf{R}_0$ or a T_1 space. Then*

$$\pi w((2^X, \tau_V)) = \pi\chi((2^X, \tau_V)) = \pi w(X).$$

For $\Delta = \mathcal{C}(X)$, Theorem 3.1 implies the following result:

THEOREM 3.3. *Let X be either a quasi-regular and $\mathbf{R}_0$ or a T_1 space. Then*

$$\pi w((2^X, \tau_F)) = \pi\chi((2^X, \tau_F)) = \max\{\pi w(X), \tau k(\mathcal{C}(X))\}.$$

Combining Theorem 3.3 and Theorem 8.7 of [9], we get the following corollary concerning the set Q of the retional numbers.

COROLLARY 3.4. $\pi w((2^Q, \tau_F)) = \pi\chi((2^Q, \tau_F)) > \aleph_0$.

If we put $\Delta = CL(X)$ and consider only the π-weight, then we obtain the following result which is similar to Theorem 3.2 but under a weaker assumption.

THEOREM 3.5. *Let X be a $\mathbf{R}_0$ space. Then*

$$\pi w((2^X, \tau_V)) = \pi w(X).$$

Note that a similar result for τ_F fails. For example, by Corollary 3.4, we have $\pi w((2^Q, \tau_F)) > \aleph_0$ while $\pi w(Q) = \aleph_0$.

4. The proofs of Theorems

LEMMA 4.1. *Let X be an $\mathbf{R}_0$ space. Let $V_1, V_2, \cdots V_n, W$ be open subsets of X and $A \in \Delta$. If $\cap_{i=1}^{n} V_i^- \cap (A^c)^+ \subseteq W^+$, then there exists some $i \leq n$ such that $V_i \setminus A \subseteq W$.*

Proof. Suppose the contrary. Then for each $i \leq n$ we can choose $x_i \in V_i \cap (X \setminus A) \cap (X \setminus W)$. Since X is $\mathbf{R}_0$ and A is closed in X, for each $i \leq n$, we have $\overline{\{x_i\}} \subseteq V_i \cap (X \setminus A)$. If $E = \cup\{\overline{\{x_i\}} : i \leq n\}$, then $E \in \cap_{i=1}^{n} V_i^- \cap (A^c)^+$ while $E \notin W^+$ which is a contradiction. $\square$

LEMMA 4.2. *Let X be a T_1 space. Let $V_1, V_2, \cdots V_n, W$ be open subsets of X and $A \in \Delta$. If $\cap_{i=1}^{n} V_i^- \cap (A^c)^+ \subseteq W^-$, then there exists some $i \leq n$ such that $V_i \cap A^c \subseteq W$.*

The proof is similar to that of Lemma 4.1 and we left it to the reader.

LEMMA 4.3. *Let X be either a quasi-regular or a T_1 space.*
If $\pi w(\Delta(X)) = \lambda$, then $\pi w(X) \leq \lambda$.

Proof. Since $\pi w(\Delta(X)) = \lambda$, we may assume that $\{\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ : \alpha \in \lambda\}$ is a π-base for $\Delta(X)$ where $V_i(\alpha)$ are open sets in X, $A \in \Delta$, and n_α is a natural number. Since $\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ \neq \emptyset$ for each $i \leq n_\alpha$, $V_i(\alpha)$ hits A_α^c. We claim that

$$\mathcal{V} = \cup\{\{V_1(\alpha) \setminus A_\alpha, V_2(\alpha) \setminus A_\alpha, \cdots, V_{n_\alpha}(\alpha) \setminus A_\alpha\} : \alpha \in \lambda\}$$

is a π-base for X.

Now let us take a non-empty open subset G of X. We will look for an element $V \in \mathcal{V}$ which is contained in G. We distinguish the following two cases.

Case 1. X is T_1.

As G is non-empty and X is T_1, G^- is a non-empty open subset of $\Delta(X)$. Therefore there exists $\alpha \in \lambda$ such that $\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ \subseteq G^-$. By Lemma 4.2, there exists $i \leq n_\alpha$ such that $V_i(\alpha) \setminus A_\alpha \subseteq G$. Hence, $V = V_i(\alpha) \setminus A_\alpha$ is as required.

Case 2. X is quasi-regular.

As G is non-empty, it contains a non-empty closed subset E whose interior $intE$ is non-empty. It is obvious that $(intE)^-$ is a non-empty open subset of $\Delta(X)$. Therefore there exists $\alpha \in \lambda$ such that $\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ \subseteq (\ intE)^-$. We will show that there is $i \leq n_\alpha$ such that $V_i \setminus A_\alpha \subseteq G$. If not, then for each i, we have that $V_i \cap A_\alpha^c \cap G^c \neq \emptyset$. Thus $V_i \cap A_\alpha^c \cap E^c \neq \emptyset$. By the previous argument, there are non-empty closed subsets F_i such that $F_i \subseteq V_i(\alpha) \cap A_\alpha^c \cap E^c$. If we set $F = \cup\{F_i : i \leq n_\alpha\}$, then $F \in \cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+$, but $F \notin (\ intE)^-$ which is a contradiction. □

LEMMA 4.4. *Let X be an $\mathbf{R}_0$ space. If $\pi w(\Delta(X)) = \lambda$, then $\tau k(\Delta) \leq \lambda$.*

Proof. Let $\{\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ : \alpha \in \lambda\}$ be a π-base for $\Delta(X)$, where $V_i(\alpha)$ are open sets in X, $A_\alpha \in \Delta$, and n_α is a natural number. We claim that

$$\mathcal{B} = \{A_\alpha : \alpha \in \lambda\}$$

is a π-cofinal subset of Δ. Take an open subset G of X and $A \in \Delta$ such that $G \cap A^c \neq \emptyset$. Since X is $\mathbf{R}_0$, it follows that $G^- \cap (A^c)^+$ is a non-empty open subset of $\Delta(X)$. Therefore there exists $\alpha \in \lambda$ such that $\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ \subseteq G^- \cap (A^c)^+$. If we take $E \in \cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+$, then $E \cap G \neq \emptyset$ and $E \cap A_\alpha = \emptyset$. Consequently, G hits $(A_\alpha)^c$. We demonstrate that A_α contains A. In fact, if not, we can take $a \in A \setminus A_\alpha$. As X is $\mathbf{R}_0$, one has $\overline{\{a\}} \subseteq A_\alpha^c$. If $F = \overline{\{a\}} \cup E$, then $F \in \cap_{i=1}^{n_\alpha} V_i^- \cap (A_\alpha^c)^+$, but $F \notin (A^c)^+$ which is a contradiction. □

LEMMA 4.5. *Let (X, τ) be a pseudo-$\mathbf{R}_0$ space.*
If $\max\{\pi w(X), \tau k(\Delta)\} = \kappa$, then $\pi w(\Delta(X)) \leq \kappa$.

Proof. Let $\mathcal{V} = \{V_\alpha : \alpha \in \kappa\} \subseteq \tau \setminus \{\emptyset\}$ be a π-basis of (X, τ), and $\mathcal{A} = \{A_\alpha : \alpha \in \kappa\} \subseteq \Delta$ a τ-cofinal subset of Δ. By the condition (b), without loss of the generality, we may assume that $\mathcal{A}$ is closed with respect to finite (not necessarily non-empty) intersections. Denote by $\kappa^{<\omega}$ the family of all finite subsets of κ. Define

$$\mathbf{W} = \{\cap_{\beta \in E} V_\beta^- \cap (A_\alpha^c)^+ : \alpha \in \kappa, E \in \kappa^{<\omega}\}.$$

It is obvious that the cardinality of $\mathbf{W}$ is κ. To complete the proof, it suffices to show that every non-empty open subset of $\Delta(X)$ contains some non-empty element of $\mathbf{W}$. Let $\mathcal{G}$ be a non-empty open subset of $\Delta(X)$.

We may assume that, without loss of the generality, $\mathcal{G} = \cap_{i=1}^{n} G_i^- \cap (A^c)^+$ where G_i's are non-empty open subsets of X and $A \in \Delta$. Since $\mathcal{G}$ is non-empty, it follows that $G_i \cap A^c \neq \emptyset$ for every $i \leq n$. Since $\mathcal{V}$ is a $\pi-$basis of (X, τ), for each $i \leq n$ there exists $\alpha_i \in \kappa$ such that $V_{\alpha_i} \subseteq G_i \cap A^c$. Now consider all such pairs (V_{α_i}, A) for all $i \leq n$. Since $\mathcal{A}$ is $\tau-$cofinal in Δ, for each $i \leq n$ there exists $\beta_i \in \kappa$ such that $A_{\beta_i}^c$ hits V_i and that A_{β_i} contains A. We set $A' = \cap\{A_{\beta_i} : i \leq n\}$. It is straightforward to verify that $A \subseteq A'$ and V_{α_i} hits $(A')^c$ for all $i \leq n$. Let $\mathcal{W} = \cap_{i=1}^{n} V_{\alpha_i}^- \cap ((A')^c)^+$. It is easy to see that $\mathcal{W} \in \mathbf{W}$ and that $\mathcal{W}$ is non-empty. It is not difficult to verify that $\mathcal{G}$ contains this $\mathcal{W}$. □

The proof of Theorem 3.5. Accoding Lemma 4.5 and Corollary 2.4, we have $\pi w((2^X, \tau_V)) \leq \pi w(X)$. Thus we only need to show that $\pi w((2^X, \tau_V)) \geq \pi w(X)$. Now let $\pi w((2^X, \tau_V)) = \lambda$. We may assume that $\{\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ : \alpha \in \lambda\}$ is a π-base for $(2^X, \tau_V)$ where sets $V_i(\alpha)$ are open in X, A_α is closed in X and n_α is a natural number. Since $\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ \neq \emptyset$ for each $i \leq n_\alpha$, V_i hits A_α^c. We claim that

$$\mathcal{V} = \cup\{\{V_1(\alpha) \setminus A_\alpha, V_2(\alpha) \setminus A_\alpha, \cdots, V_{n_\alpha}(\alpha) \setminus A_\alpha\} : \alpha \in \lambda\}$$

is a π-base for X.

Now let us take a non-empty open subset G of X, we will look for an element $V \in \mathcal{V}$ which is contained in G.

The case $G = X$ is trivial. Hence, we assume that $G \neq X$.

Let $A = X \setminus G$. Then $G^+ = (A^c)^+ \in \tau_V$ and it is non-empty. Therefore there exists $\alpha \in \lambda$ such that $\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ \subseteq G^+$. By Lemma 4.1, there exists $i \leq n_\alpha$ such that $V_i(\alpha) \setminus A_\alpha \subseteq G$. If we take $V = V_i(\alpha) \setminus A_\alpha$, then this V is the required one. □

LEMMA 4.6. *Let X be either a quasi-regular and $\mathbf{R}_0$ or a T_1 space. If $\pi\chi(X, \Delta(X)) = \kappa$, then $d(X) \leq \kappa$, where $d(X)$ means the density of X.*

Proof. Without loss of the generality, we can assume that

$$\mathbf{W} = \{\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ : \alpha \in \kappa\}$$

is a local π-basis for $\Delta(X)$ at this point X where sets $V_i(\alpha)$ are open in X, $A_\alpha \in \Delta(X)$, and n_α is a natural number. Since for each $\alpha \in \kappa$,

$\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ \neq \emptyset$, $V_i(\alpha)$ hits A_α^c for all $i \leq n_\alpha$ and $\alpha \in \kappa$. Take $a_{i\alpha} \in V_i(\alpha) \cap A_\alpha^c$ and, let $D = \{a_{i\alpha} : \alpha \in \kappa, i \leq n_\alpha\}$. We show that D is a dense subset of X.

Let V be a nonempty open subset of X. Thus $X \in V^-$. Since $\mathbf{W}$ is a local $\pi-$ basis for $\Delta(X)$ at this point X, there is $\alpha \in \kappa$ such that $\cap_{i=1}^{n_\alpha} V_i(\alpha)^- \cap (A_\alpha^c)^+ \subseteq V^-$. By Lemma 4.2, there is some $i \leq n_\alpha$ such that $V_i(\alpha) \cap A_\alpha^c \subseteq V$. Hence $a_{i\alpha} \in D \cap V$. □

The proof of Theorem 3.1. By Lemma 4.3, 4.4, and 4.5, it follows that $\pi w(\Delta(X)) = \max\{\pi w(X), \tau k(\Delta)\}$. By Lemma 4.6, it follows that $d(X) \leq \pi\chi(\Delta(X))$. A straightforward check shows that $d(X) \geq d(\Delta(X))$. Hence, $d(\Delta(X)) \leq \pi\chi(\Delta(X))$. According to Theorem 3.8 1(b) of [7], one has

$$\pi w(\Delta(X)) = \max\{\pi\chi(\Delta(X)), d(\Delta(X))\} = \pi\chi(\Delta(X)).$$

Consequently, $\pi w(\Delta(X)) = \pi\chi(\Delta(X)) = \max\{\pi w(X), \tau k(\Delta)\}$. □

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