ON CR-SUBMANIFOLDS OF A CERTAIN CLASS OF ALMOST CONTACT MANIFOLDS (*)

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SOMMARIO. - *Nel 1978, A. Bejancu introdusse e studiò le sotto varietà CR di una varietà Kahleriana* [1]. *Da allora molti articoli sono apparsi su questo argomento, che è ancora oggi il soggetto di molte ricerche. D'altra parte Kenmotsu* [7] *ha studiato una nuova classe di varietà Riemanniane di quasi-contatto, note come varietà di Kenmotsu. Le sotto varietà semi-invarianti di tali varietà sono state studiate da Kobayashi* [10] *e Papaghuice* [11]. *L'obiettivo di questo articolo è di continuare questo studio e ottenere dei nuovi risultati su questo argomento.*

SUMMARY. - *In 1978, A. Bejancu introduced and studied CR-submanifolds of a Kaehler manifold* [1]. *Since then many papers appeared on this topic. On the other hand Kenmotsu* [6] *studied a new class of almost contact Riemannian manifolds, known as Kenmotsu manifolds. Semi-invariant submanifolds of such manifolds were studied by Kobayashi* [8] *and Papaghuice* [10]. *The purpose of this paper is to continue this study and to obtain some more results on this topic.*

1. Introduction.

Let $\bar{M}$ be a $(2n+1)$-dimensional almost contact metric manifold with structure tensors (ϕ, ξ, η, g) where ϕ is a tensor field of type $(1,1)$, ξ is a vector field, η is a 1-form and g is the Riemannian metric on $\bar{M}$. These tensors satisfy [3]

$$\phi^2 X = -X + \eta(X)\xi, \;\; \phi\xi = 1, \;\; \eta(\xi) = 1, \;\; \eta(\phi X) = 0 \tag{1.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{1.2}$$

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for any vector field X, Y tangent to $\bar{M}$. We denote by $\bar{\nabla}$ the covariant differentiation with respect to the metric g on $\bar{M}$. Then it is known that $\bar{M}$ is a Kenmotsu manifold if

$$(\bar{\nabla}_x\varphi)(Y) - \eta(Y)\phi X - g(X, \phi Y)\xi \tag{1.3}$$

$$\bar{\nabla}_X\xi = X - \eta(X)\xi \tag{1.4}$$

for any X, Y tangent to $\bar{M}$.

This structure is closely related to the wraped product of two Riemannian manifold. one of the typical example of Kenmotsu manifold is the hyperbolic space $\bar{M}(-1)$.

Now, let M be an n-dimensional isometrically immersed submanifold of $\bar{M}$. We assume that ξ is always tangent to M.

For a vector field x tangent to M we put

$$\phi X = PX + QX \tag{1.5}$$

where PX (resp. QX) is the tangential (resp. normal) component of ϕX.

For a vector field N normal to M we put

$$\phi N = tN + fN \tag{1.6}$$

where tN (resp. fN) is th tangential (resp. normal) component of ϕN.

DEFINITION. An n-dimensional Riemannian submanifold M of a Kenmotsu manifold $\bar{M}$ is called a CR-submanifold if ξ is tangent to M and there exists on M a differentiable distribution $D : x \to D_x \subset T_xM$. Satisfying the following conditions:

(i) D_x is invariant under ϕ i.e. $\phi D_x \subset D_x$ for each $x \in M$, and (ii) the complementary orthogonal distribution $D^\perp : x \to D_x^\perp \subset T_xM$ is totally real i.e. $\phi D_x^\perp \subset T_x^\perp M$ for each $x \in M$.

If $\dim D_x^\perp = 0$ (resp. $D_x = 0$) then the CR-submanifold is called an invariant (resp. totally real) submanifold. The pair $(D, D^\perp)$ is called ξ-horizontal (resp. ξ-vertical) if $\xi_x \in D_x$ (resp. $\xi_x \in D_x^\perp$) for each $x \in M$ [8]. A CR-submanifold is said to be proper if it is neither an invariant nor an anti-invariant submanifold.

Now, the Gauss and Weingarten formulas are respectively given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X,Y) \tag{1.7}$$

$$\bar{\nabla}_X N = A_N X + \nabla^{\perp}_X N \tag{1.8}$$

where $\bar{\nabla}$ is the Riemannian connection of $\bar{M}$, ∇ the Riemannian connection determined by the induced metric g on M, $\nabla^{\perp}$ the metric connection in the normal bundle of M, and h and A are both second fundamental tensors satisfying

$$g(h(X,Y),N) = g(A_N X, Y) \ . \tag{1.9}$$

DEFINITION. A CR-submanifold M of Kenmotsu manifold $\bar{M}$ is said to be D-totally geodesic (resp. $D^{\perp}$-totally geodesic) if $h(X,Y) = 0$ for $X, Y \in D$ (resp. $h(W,Z))$=0 for all $W, Z \in D^{\perp}$.

The equation of Gauss is given by

$$\bar{R}(X,Y,Z,W) =$$

$$R(X,Y,Z,W) - g(h(X,Z),h(Y,Z)) + g(h(Y,W),\ h(X,Z)) \tag{1.10}$$

where $\bar{R}$ (resp. R) be the curvature tensor of $\bar{M}$ (resp. M).

A Kenmotsu manifold $\bar{M}$ with constant ϕ-sectional curvature c is called Kenmotsu space form denoted by $\bar{M}(c)$ and the curvature tensor is given by [7].

$$\begin{aligned}\bar{R}(X,Y,Z,W) = {} & (c-3)/4(g(Y,Z)g(X,Z) - g(X,Z)g(Y,W)) \\ & +(c+1)/4\{\eta(X)\eta(Z)g(Y,W) - \eta(Y)\eta(Z)g(X,W) \\ & +\eta(Y)\eta(W)g(X,Z) - \eta(X)\eta(W)g(Y,Z) \\ & +g(X,\phi Z)g(\phi Y,W) - g(Y,\phi Z)g(\phi X,W) \\ & +2g(X,\phi Y)g(\phi Z,W)) \end{aligned} \tag{1.11}$$

for any $X, Y, Z, W \in TM$.

2. Basic Lemmas for a CR-submanifold in a Kenmotsu Manifold.

We start with

LEMMA 2.1. *Let M be a CR-submanifold of a Kenmotsu manifold $\bar{M}$. Then we have*

$$P\nabla_X PY - PA_{QY}X = P\nabla_X Y - \eta(Y)PX - g(X,PY)P\xi \tag{2.1}$$

$$\begin{aligned} Q\nabla_X PY - QA_{QY}X = Q\nabla_x Y - \eta(Y)QX \\ -\mathrm{th}(X,Y) - g(X,PY)Q\xi \end{aligned} \tag{2.2}$$

$$h(X,PY) + \nabla^{\perp}_X QY = fh(X,Y) \tag{2.3}$$

for any X, Y tangent to M.

Proof. The proof follows immediately from (1.3) by equating horizontal, vertical and normal component.

LEMMA 2.2. *Let M be a CR-submanifold of Kenmotsu manifold $\bar{M}$. Then we have*

$$A_{QW}Z = A_{QZ}W$$

for any $W, Z \in D^{\perp}$.

Proof. From (1.7)–(1.9) and and using (1.3) for $Y \in TM$ we get

$$\begin{aligned} g(A_{QW}Z,Y) &= g(h(Y,Z),\phi W) = g(\nabla_Y Z, \phi W) \\ &= -g(\phi\nabla_Y Z, W) = -g(\nabla_Y \phi Z, W) \\ &= g(A_{\phi Z}Y, W) = g(A_{\phi Z}W, Y) \end{aligned}$$

from which the assertion follows:

DEFINITION. In a CR-submanifold of a Kenmotsu manifold we define

$$(\bar{\nabla}_X P)(Y) = \nabla_X PY - P\nabla_X Y \tag{2.4}$$

$$(\bar{\nabla}_X Q)(Y) = \nabla^{\perp}_X QY - Q\nabla_X Y \tag{2.5}$$

$$(\bar{\nabla}_X t)(N) = \nabla_X tN - t\nabla^{\perp}_X N \tag{2.6}$$

$$(\bar{\nabla}_X f)(N) = \nabla^{\perp}_X fN - f\nabla^{\perp}_X N \tag{2.7}$$

for any vector field X and Y tangent to M and any vector field N normal to M.

DEFINITION. The endomorphism P (resp. the endomorphism f, the 1-forms Q and t) is parallel if $\nabla P = 0$ (resp. $\nabla f = 0$, $\nabla Q = 0$ and $\nabla t = 0$).

By using (1.3) and (1.5)–(1.8) we easily have the following:

$$\begin{aligned} (\bar{\nabla}_X P)(Y) &= A_{QY}X - \eta(Y)PX - g(X,PY)\xi + th(X,Y)) && (2.8)\\ (\bar{\nabla}_X Q)(Y) &= fh((X,Y) - h(X,PY) - \eta(Y)QX\,, && (2.9)\\ (\bar{\nabla}_X t)(N) &= A_{fN}X - PA_NX + g(\phi X, N)\xi\,, && (2.10)\\ (\bar{\nabla}_X f)(N) &= -h(X,fN) - QA_NX && (2.11) \end{aligned}$$

for any vector field X, Y tangent to M and any vector field normal to M.

Now from (2.8), (2.9) and by virtue of (2.4) and (2.5) we have

$$P\nabla_X Z = -A_{QZ} - th(X,Z) + \eta(Z)PZ\,, \qquad (2.12)$$

$$Q\nabla_X Z = \nabla^{\perp}_X QZ - fh(X,Z) - \eta(Z)QX \qquad (2.13)$$

for any tangent to M and $Z \in D^{\perp}$.

Putting $X = Z \in D^{\perp}$ in (2.12) and using Lemma 2.2 we have

$$\begin{aligned} P(Z,W) &= P\nabla_Z W - P\nabla_W Z\\ &= -A_{QW}Z + \eta(W)PZ + A_{QZ}W + \eta(Z)PW \end{aligned}$$

for any $W, Z \in D^{\perp}$.

On the other hand (2.13) yields

$$\begin{aligned} \phi[Z,W] &= Q[Z,W] = Q\nabla_Z W - Q\nabla_W Z\\ &= \nabla^{\perp}_Z QW - \eta(W)QZ - \nabla^{\perp}_W QZ - \eta(Z)QW \end{aligned}$$

Hence we have the following

LEMMA 2.3. *Let M be a CR-submanifold of a Kenmotsu manifold $\bar{M}$. Then*

$$\nabla^{\perp}_W QZ - \nabla^{\perp}_Z QW - \eta(W)QZ - \eta(Z)QW \in \phi D^{\perp} \text{ for } W, Z \in D^{\perp}\,.$$

As a corollary of Lemma 2.3 we have the following:

COROLLARY 2.4. *Let M be a ξ-horizontal CR-submanifold of a Kenmotsu manifold $\bar{M}$. Then*

$$\nabla^{\perp}_{Z}QW - \nabla^{\perp}_{W}QZ \in \phi D^{\perp} \tag{2.14}$$

for any $W, Z \in D^{\perp}$.

3. CR-submanifold with parallel structure and contact CR-product.

In this section we shall study the case in which the canonical structure is parallel. Moreover, we give a characterization of CR-product in terms of the induced canonical structure tensor on a submanifold.

First we have

LEMMA 3.1. *Let M be a CR-submanifold of a Kenmotsu manifold $\bar{M}$. If Q is parallel then*

$$h(X, PY) = h(Y, PX) \tag{3.1}$$

for any $X, Y \in D$.

Proof. From (2.9) we obtain

$$h(X, PY) = fh(X, Y)$$

for any $X, Y \in D$. Interchanging X be Y we get (3.1)

Now we have

PROPOSITION 3.2. Let M be a proper CR-submanifold of a Kenmotsu manifold $\bar{M}$. if M is non-totally geodesic, totally umbilical then both D and $D^{\perp}$ are non integrable.

Proof. As M is totally umbilical, so we have

$$h(U, V) = g(U, V)H$$

First we assume that D is integrable. Then putting $U = X, V = PY$ for $X, Y \in D$ we have

$$h(X, PY) = h(Y, PX)$$

Since M is non-totally geodesic, then for all $X, Y \in D$ we have

$$g(X, PY) = g(Y, PX) .$$

Hence $D = 0$ which is a contradiction.Similarly, we can show that $D^{\perp}$ is non-integrable.

Next we have the following

PROPOSITION 3.3. Let M be a CR-submanifold of a Kenmotsu manifold $\bar{M}$. Then Q is parallel if and only if t is parallel.

Proof. Suppose t is parallel i.e. $\nabla t = 0$. Then from (2.10) we have

$$g(A_{fN}X, Y) = g(PA_N X, Y) - g(\phi X, N)g(Y, \xi) ,$$

for any X, Y tangent to M and any vector field N normal to M.

By virtue of (1.9) the above equation yields

$$g(fh(X, Y), N) = g(h(X, PY), N) + g(\phi X, N)g(Y, \xi)$$

from which

$$fh(X, Y) = h(X, PY) + \eta(Y)\phi X$$

Thus we get

$$\nabla Q = 0 .$$

For the canonical structure P, we have the following, the proof of which easily follows from (2.8).

LEMMA 3.4. *Let M be a CR-submanifold of a Kenmotsu manifold $\bar{M}$. Then P is parallel if*

$$A_{QX}Y - A_{QY}X = \eta(Y)PX - \eta(X)PY$$

for any X, Y tangent to M.

Now we have

PROPOSITION 3.5. *Let M be a CR-submanifold of a Kenmotsu manifold $\bar{M}$. If P is parallel then the horizontal distribution D is integrable.*

Proof. Suppose P is parallel, then (3.2) gives

$$A_{\phi Y}X = \eta(X)PY - \eta(Y)PX$$

for any $X \in D$, $Y \in TM$.

Now for $X \in D$, $Y \in D^{\perp}$ and taking an inner product with $Z \in D$, we get

$$g(A_{\phi Y}X, Z) = 0$$

which is equivalent to

$$g(h(X, Z), QY) = 0$$

for any $X, Z \in D$ and $Y \in D^{\perp}$.

DEFINITION. A CR-submanifold of a Kenmotsu manifold is called CR-product if it is locally a Riemannian product of an invariant submanifold M^T and an anti-invariant submanifold $M^{\perp}$.

The following characterization is proved in [9].

PROPOSITION 3.6. Let M be a ξ-horizontal semi-invariant submanifold of a Kenmotsu manifold. Then the following statements are equivalent:

(i) M is a CR–product

(ii) $A_{\varphi D} \perp \varphi D = \{0\}$

Now we have

THEOREM 3.7. *A ξ-horizontal CR-submanifold M of a Kenmotsu manifold $\bar{M}$ is a CR-product if and only if P is parallel.*

Proof. Suppose P is parallel, then (2.8) yields

$$A_{FY}X - \eta(Y)PX - g(X, PY)\xi + \mathrm{th}(X, Y) = 0 .$$

If we take $Y \in D$, $Z \in D^{\perp}$, $X \in TM$ and using the fact that $FY = 0$ for $Y \in D$, the above equation gives

$$\eta(Y)g(PX, Z) - g(X, PY)g(\xi, Z) + g(th(X, Y), Z) = 0$$

from which we have

$$g(A_{\phi Z}\phi Y, X) = 0$$

which is equivalent to

$$A_{\varphi D} \perp \varphi D = \{0\}. \text{ Hence } M \text{ is a } CR\text{-product} .$$

Conversely, if M is a CR-product, then it is easy to prove that the endomorphism P is parallel, which completes the proof.

4. Sectional Curvature of CR-submanifold of Kenmotsu Space Form.

Let M be a CR-submanifold of a Kenmotsu space form $\bar{M}(c)$. Then from (1.10), the curvature tensor of $\bar{M}(c)$ is given by

$$\begin{aligned} R(X,Y;Z,W) = (c-3)/4[&g(Y,Z)g(X,W) - g(X,Z)g(Y,W)] \\ &+(c+1)/4[\eta(X)\eta(Z)g(Y,W) - \eta(Y)\eta(Z)g(X,Z) \\ &+\eta(Y)\eta(W)g(X,Z) - \eta(X)\eta(W)g(Y,Z) \\ &+g(X,\phi PZ)g(W,\phi PY) - g(Y,\phi PZ)g(\phi PX,W) \\ &+2g(X,\phi PY)g(\phi PZ,W)] \\ &+g(h(X,W),h(Y,Z)) - g(Y,W),h(X,Z)) \quad (4.1) \end{aligned}$$

for any $X, Y, Z, W \in TM$.

Let $\bar{H}(X)$ be the ϕ-holomorphic sectional curvature of $\bar{M}(c)$ determined by a unit vector X and ϕX such that X is orthogonal to ξ. Then using (1.11), we have

$$\bar{H}(X) = (c-3)/4 .$$

Suppose $\bar{K}(X \wedge Y)$ be th sectional curvature of $\bar{M}(c)$ determined by orthogonal vectors X and Y, then (1.11) yields

$$\bar{K}(X \wedge Y) = (c-3)/4 + (c-1)/4[\eta(X)^2 + \eta(Y)^2 - g(X,\phi Y)^2]. \quad (4.2)$$

Moreover if $K(X \wedge Y)$ denote the sectional curvature of M determined by orthonormal tangent vectors $\{X, Y\}$ of M. Then from (4.2) and by virtue of (1.10) we have

PROPOSITION 4.1. Let M be a CR-submanifold of a Kenmotsu space form $\bar{M}(c)$. The sectional curvature of M determined by orthonormal tangent vector $\{X, Y\}$ is given by

$$\begin{aligned} K(X \wedge Y) &= (c-3)/4 + (c-1)/4[\eta(X)^2 + \eta(Y)^2 - g(X, \phi PY)^2] \\ &\quad + g(h(X,X), h(Y,Y)) - \|h(X,Y)\|^2 , \end{aligned} \tag{4.3}$$

for any vector field X, Y tangent to M.

From this we easily obtain:

PROPOSITION 4.2. Let M be a CR-submanifold of a Kenmotsu space form $\bar{M}(c)$. If M is totally geodesic in $\bar{M}(c)$, then the sectional curvature of M is given by

$$K(X \wedge Y) = (c-3)/4 + (c-1)/4[\eta(X)^2 + \eta(Y)^2 - g(X, \phi PY)]$$

for any X, Y tangent to M.

PROPOSITION 4.3. Let M be a CR-submanifold of a Kenmotsu space form $\bar{M}(c)$. If M is $D^{\perp}$-totally geodesic in $\bar{M}(c)$. Then the sectional curvature of M is given by

$$K(X \wedge Y) = (c-3)/4 + (c-1)/4[\eta(X)^2 + \eta(Y)^2]$$

for any $X, Y \in TM$.

COROLLARY 4.4. *Let M be a ξ-horizontal CR-submanifold of a Kenmotsu space form $\bar{M}(c)$. If M is $D^{\perp}$-totally geodesic in $\bar{M}(c)$, then the sectional curvature of M is given by*

$$K(X \wedge Y) = (c-3)/4 \text{ for any } X, Y \in D^{\perp} .$$

PROPOSITION 4.5. Let M be a ξ-vertical CR-submanifold of a Kenmotsu space form $M(c)$. The sectional curvature of M determined by $X, Y \in D$ is given by

$$\begin{aligned} K(X \wedge Y) &= (c-3)/4 - (c-1)/4g(X, \phi PY)^2 \\ &\quad + g(h(X,X),\ h(Y,Y)) = -\|h(X,Y)\|^2 . \end{aligned}$$

PROPOSITION 4.6. Let M be a ξ-vertical CR-submanifold of a Kenmotsu space form $\bar{M}(c)$. If M is D-totally geodesic in $\bar{M}(c)$ then the sectional curvature of M is given by

$$K(X \wedge Y) = (c-3)/4 - (c-1)/4g(X, \phi PY)$$

for any $X, Y \in D$.

PROPOSITION 4.7. Let M be a ξ-horizontal CR-submanifold of a Kenmotsu space form $\bar{M}(c)$. Then the sectional curvature of M determined by $X, Y \in D^{\perp}$ is given by

$$K(X \wedge Y) = (c-3)/4 + g(h(X,X),\ h(Y,Y)) - ||h(X,Y)||^2 \,. \quad (4.4)$$

DEFINITION. The ϕ-sectional curvature of M determined by a unit vector $X \in D$ orthogonal to ξ is the sectional curvature determined by X and ϕX.

PROPOSITION 4.8. The ϕ-sectional curvature determined by $X \in D$ orthogonal to ξ is given by

$$H(X) = (c-3)/4 + 4g(X, \phi PX)^2 + g(h(X,X),\ h(\phi X, \phi X)) - ||h(X, \phi X)||^2 \,. \quad (4.5)$$

LEMMA [9]. *Let M be a semi-invariant submanifold of a Kenmotsu manifold $\bar{M}$. Then D is integrable if and only if*

$$h(X, \phi X) = h(Y, \phi X) \quad (4.6)$$

for any $X, Y \in D$.

Using (4.5) and (4.6) we have

PROPOSITION 4.9. Let M be a CR-submanifold f a Kenmotsu space form $\bar{M}(c)$. If D is involutive satisfying $g(X, PX) = 0$ then

$$H(X) \leq (c-3)/4$$

where $H(X)$ is the ϕ-sectional curvature determined by $X \in D$ orthogonal to ξ.

Also from (4.5) we have

PROPOSITION 4.10. Let M be a CR-submanifold of Kenmotsu space form $\bar{M}(c)$. Then M is D-totally geodesic if and only if the following conditions are satisfied.

(i) the horizontal distribution D is involute, ii) $H(X) = (c-3)/4$.

Let $\{E_0 = \xi, E_1, E_2, \dots E_{m-1}\}$ be a local field of orthonormal frames on M such that in the case when Mm is ξ-horizontal (resp. ξ-vertical) $(E_0 = \xi,\ E_1, \dots, E_{p+1} = \phi E_1, \dots, E_{2p} = \phi E_{2p})$ (resp.) $(E_1, \dots E_p, E_{p+1} = \phi E_1, \dots \phi E_{2p} = \phi E_p\}$ is a local frame field on D. A ξ-horizontal CR-submanifold M is called D-minimal (resp. $D^\perp$-minimal) if $\sum_{i=1}^{2p} h(E_i, E_i) = 0$ (resp. $\sum_{f=1}^{-2p} h^{-1}(E_{2p+j}, E_{2p+j}) = 0)$.

Finally we prove

PROPOSITION 4.11. Let M be a $D^\perp$-minimal ξ-horizontal CR-submanifold of a Kenmotsu space form $\bar{M}(c)$. Then M is $D^\perp$-totally geodesic if and only if

$$K(X, Y) = (c-3)/4 \tag{4.7}$$

for any $X, Y \in D^\perp$.

Proof. For any $X, Y \in D^\perp$, (4.4) gives

$$K(X \wedge Y) = 1/4(c-3) + g(h(X,X),\ h(Y,Y)) - ||h((X,Y)||^2$$

Suppose (4.7) holds, then using minimality condition we have

$$\sum_{i,j=1}^{m-2p-1} h(E_{2p+1'} E_{2p+j})|| = 0$$

which gives $h(X, Y) = 0$ for all $X, Y \in D^\perp$. Hence M is $D^\perp$-totally geodesic.

Similarly we have

PROPOSITION 4.12. Let M be a D-minimal ξ-vertical CR-submanifold of a Kenmotsu space form $\bar{M}(c)$. Then M is D-totally geodesic if and only if

$$K(X \wedge Y) = (c-3)/4$$

for any $X, Y \in D$ with $g(\phi X, Y) = 0$.

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