DYNAMIC FACTOR IN THE MECHANICS OF FRACTURE IN BRITTLE MATERIALS (*)

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SOMMARIO.- *Le forze interne che si manifestano in connessione con la dinamica del processo di danneggiamento, influenzano l'estensione del danno, sovrapponendosi alla azione delle forze esterne nel produrre la frattura.*
Il fattore dinamico del processo costituisce una delle cause del fenomeno che rivela la dipendenza della resistenza alla rottura dei materiali friabili e della loro energia dalla deformazione e dal metodo di rinforzo della deformazione.
La descrizione del fenomeno è stata fondata sulla base dei concetti usati nella teoria statisca della resistenza e nella teoria dei danni e delle onde di stress.

SUMMARY.- *Internal forces due to arise in connection with the dynamic procedure of the process of damage influence the extension of damage, assisting the external forces in their action on producing fracture.*
Dynamic factor of the process, linked with the neutralizing factors affecting those dynamics constitutes one of causes of the phenomenon, that reveals the dependence of the tensile strength of brittle materials and their energy of fracture on the strain rate and method of deformation enforcement.
The description of the phenomenon has been set up on the base of conceptions used in the statistic theory of strength, damage theory and stress waves theory.

1. INTRODUCTION.

Facts ascertained at various levels of experimental observation confirm the knowledge that development of fracture proceeds by abrupt rises and is characteristic for certain dynamics. Despite that, in the analyses of the mechanism of fracture, the effect of dynamic factor is neglected.

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The first attemp to formulate this problem, regarding relations coming from the classical mechanics, was undertaken by author in the paper [1]. In the present publication these topics have been developed. In particular the paper contains:

— more exact description of the model of the brittle material, which takes into consideration in general terms the conditions in which the dynamics effects connected with damage are created and comulated,

— analysis and systematic division of the processes concerning the influence of strain rate on strength,

— more exact description of the low rate processes with dynamic procedure, covering the phase of falling branch of the σ - ε relation.

2. MODEL.

2.1. Assumptions and structural features.

Usually the theoretical problems concerning the fracture of the brittle materials are solved with a reference to the model of a crack. However in the course of its development a crack encounters on its way obstacles and there are to be noted then a temporary stoppage in the development of crack. The further propagation of crack in condition of increasing loading can take place only when the said obstacles are succesfully broken up. Those incidents are always connected with dynamic effects, which are difficult to be duly accounted for in a model of a crack.

Consequently having in mind a setting up of an investigation on the dynamics in a destruction process, the author decided to adopt a model whose features are linked with the notions originated in the theory of damage. This theory deals with the fact of the dynamic procedure of the process implicitly.

At the contruction of the model it has been assumed that under the action of tensile forces the material takes the form of a structure of multigrade freedom consisting of discreet, interchanged mass and contact layers (fig. 2.1).

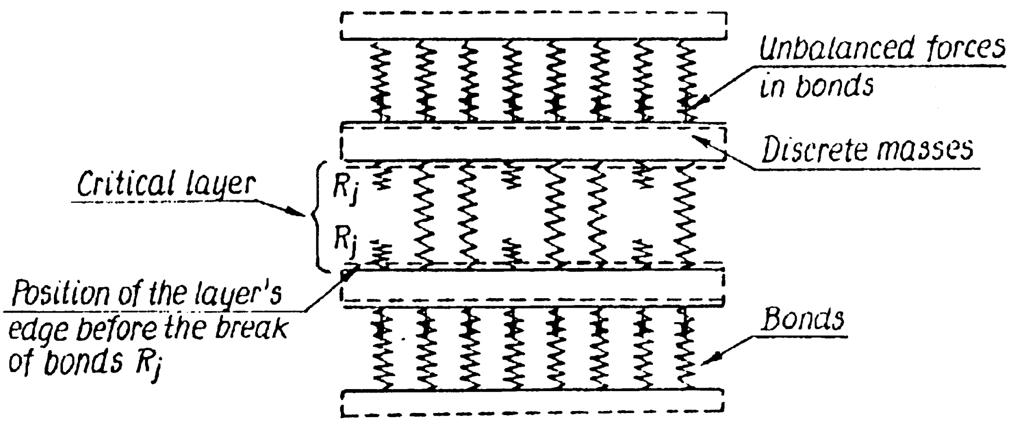

Fig. 2.1

All bonds of the contact layers have identical elasticity moduls and behave according to the rules of elasticity until destruction takes place. Their individual strength varies however. That is why the process of damage in the contact layers will mount up successively, consistently with the state of deformation of the model.

Considering the distribution of strength the systems of bonds in the particular layers are similar but, not identical. That is why the process of damage in each layer will develop slightly differently. The weakest system of bonds can be observed in the critical contact layer. There, in the future, the plane of the fracture in the model will arise.

2.2. Description of contact layers.

It has been assumed that the strength of bonds belonging to any of the contact layers of the model are determined by means of a distribution function $\varphi_i(R)$, where "R" denotes the strength of individual bonds, and "i" the ordinal number of a particular layer.

Functions $\varphi_i(R)$ are continuous, determined within certain ranges of strength $<R_{i\,min}, R_{i\,max}>$. They supply the information to the question what part "P_j" of a given series of bonds forming the particular contact layers constitute bonds of the strength R_j

$$P_{ij} = \left[\varphi_i\left(R\right)\right]_{R=R_j} \Delta R. \tag{2.1}$$

Here, ΔR = constant, represents the increment equal to the difference between two successive classes of strength ($\Delta R = R_2 - R_1 = R_3 - R_2$ etc.).

The increment ΔR, expresses at the same time the unit of measurement of the R_j - strength of bonds and the latter can be presented in the form of the product

$$R_j = j \, \Delta R \tag{2.2}$$

where "*j*" is the ordinary number subordinated to the given class of strength, defining its place in the hierarchy of the set.

The number of classes in a set "*i*" equals n_i. The highest strength therefore

$$R_{i\,max} = n_i \, \Delta R \, . \tag{2.3}$$

The increment ΔR represents a quantity finite yet very small. We are entitled therefore to treat ΔR, in certain conditions, as dR i.e. the increment infinitely small and to apply to the problem the rules of integration. Thus the total cross-sectional area of the undamaged layer is

$$\int_{R_{i\,min}}^{R_{i\,max}} \varphi_i \,(R)\, dR = 1 \, . \tag{2.4}$$

The extent of damage corresponding to a certain stage "*j*" of the process

$$u_{ij} = \int_{R_{i\,min}}^{R_j} \varphi_i \,(R)\; dR \, . \tag{2.5}$$

Accordingly, the nominal stresses at this stage of the process

$$\sigma_{ij} = R_j \left(1 - u_{ij}\right) . \tag{2.6}$$

Either n_i - the number of classes of strength or the number of bonds belonging to the individual classes of strength are large figures. The filling up of the contact layer with bonds belonging to particular classes of strength is therefore uniform. Due to that, in a case of an axially-symmetrical tensile force applied, the extent of damage will grow up at a uniform rate, and the model will be inwardly balanced at all stages of the process.

3. DYNAMICS OF THE FRACTURE.

3.1. Carry-over effect and temporary stresses.

We consider the process of damage taking place in one of the contact layers. After the bonds of each successive class of strength R_j are broken, the load which had been carried by them is taken over by the remaining unbroken bonds; this phenomenon shall be called farther on "the carry-over effect".

In connection with the carry-over effect, in the damaged layer the permanent increment of the actual stresses Δs_{rj} takes place. Its value can be calculated from the relation

$$\Delta s_{rj} = \frac{[\varphi_i (R)]_{R = R_j} \Delta R \, R_j}{\int_{R_j}^{R_{imax}} \varphi_i \; (R) \; dR} . \tag{3.1}$$

The numerator of the fraction of the above relation represents the sum of forces which have been carried by the bonds of the class R_j, while the denominator expresses the area of the surface of the contact layer with the extent of damage duly accounted for.

If the carry - over effect proceeded slowly then, in the result of a permanent increment Δs_{rj} an increment of elongation of the critical layer and, equal to it the displacement of the bottom part of the model and the load suspended on it, would only take place. In fact however, the carry-over effect proceeds suddenly and, in the course of its development, there are generated in the contact layer the temporary deformations and the temporary stresses.

The mechanism of the development of the temporary stresses runs along the following line. At the moment when in the analised i-contact layer the break of bonds class R_j takes place, in the two contact layers nearest to the i- layer arise suddenly unbalanced forces (fig. 2.1) which tend to shift mass- layers situated at the edges of the i-contact layer. Further on, the action of these forces shall be called "the group impulse".

In the case a model is subjected to the axial tension, the group impulse forces are uniformly distributed over the area of the cross-sections of both contact layers adjacent to the one where the damage has arisen.

Under the action of the group impulse, the inertia forces in both discrets mass layers situated at the edges of the i-contact layer will arise. At the very beginning they constitute the only reaction that counterbalances the forces of the group impulse.

Gradually, as the inertia forces are being overcome by the forces of the group impulse, the discrete masses at the edges of the i-contact layer are gaining an acceleration and, their displacement causes a gradual relaxation of the forces of group impulse (which are located in bonds of contact layers adjacent to the one where the demage has arisen). In that way the state of unbalanced impulse force will proceed gradually into the successive contact layers, giving rise to the wave motion in which all mass layers are taking part.

The movement of the discrete masses at the edges of the i-contact layer is restrained by the bonds of this layer. Consequently according as the component of the reaction against impulse originated from the inertia forces will diminish, the component of the reaction due to the i-layer bonds, will increase. Finally, the carry-over effect will by entirely completed and, in the bonds of the i-contact layer an increment of the actual tensile stresses, equal Δs_{rj} will be noted. (The value of this increment can be calculated from the formula (2.1)).

At the moment when in the i-layer the carry-over effect developes, in the bonds of the two adjacent contact layers, the temporary compressive stresses are generated. The maximum absolute value of these stresses is naturally equal- Δs_{rj}.(1)

The proposed model does not contain any implications concerning the quantitative estimations of the discrete masses. By reducing them at will we can shorten periods of their self-vibrations and the velocities of the wave- motion in a dicrete matter. Thus we can achieve the conditions which this motion can be treated as the wave-motion of a continuous matter and investigated in the category of stress-waves.

3.2. Mounting up of increments of the permanent and temporary stresses in effect of the detached group impulses.

The process of damage developing in the model in conditions of the low rate loading increase in being considered. It has been assumed that the edge- conditions of the model corresponds to one of the schemes generally applied in the course of tests of tensile strength carried out on the brittle materials (fig. 3.1).

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(1) The figure illustrating the results of the foregoing analysis have been given in the paper [1] p. 84, fig. 3. ("Development of the reaction that counterbalances the force of the group impulse")

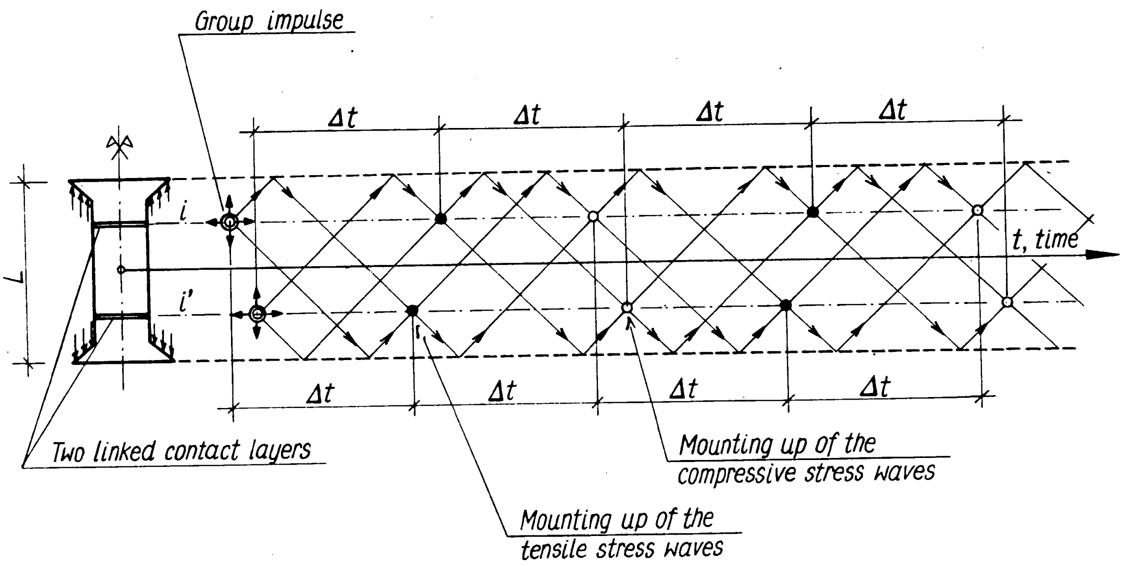

Fig. 3.1

In those circumstances two waves of compressive stresses - wave No 1 and wave No 2, developed at the edges of the *i*-contact layer, falling upon the and faces of the model and, reflected there, change the phase giving rise to two waves of the tensile stresses, and so on.

The process of fracture develops simultaneously in all contact layers, though not with the same force in each one. Still, in all of them, group impulses arise and stress-waves are generated.

Considering the development of the destruction process, there have to be classified out as essential ones the moments, at which both tensile stress- waves (i.e., wave No 1 and No 2) attain the maximum value of the mounting-up. On analysing that problem in a case of a contact layer "*i*", chosen at random, it has to be noted that the said mounting-up develops on the area of the layer "*i'*", situated symmetrically in respect of the layer "*i*" where the stress-waves have been generated (fig. 3.1).

The process of damage develops also in the layer "*i'*", where the stress-waver are also emitted. They mount up as tensile stress waves on the area of the layer "*i*". So, it can be deduced from the foregoing consideration that each pair of contact layers situated simmetrically in respect of the central plane of the model, is linked by interaction consisting in the mutual bringing about a mounting up of tensile stresses. Further on, this pair of layers situated symmetrically in respect of the central plane of the model shall be referred to as a "linked layers". (2)

Thus, in the damage process developing dynamically, a number of pairs of linked layers and one central layer are taking part. (The tensile stress-waves which mount up in the latter have been caused by the process of demage developing in it).

In figure 3.1 it can be seen that Δt-the time span between the moment of arising of stress-waves in the layer "i" and the moment of their mounting up as tensile stress-waves on the area of "i'" layer is

$$\Delta t = \frac{L}{c} \qquad (3.2)$$

where: L - length of a model, c - velocity of trevelling of the acoustic waves in the particular body.

The maximum increment of the temporary stresses (in actual stresses) appearing in the contact layer in consequence of the breakage of bonds group class R_j attain the value

$$\Delta s_j = 2\Delta s_{rj} \qquad (3.3)$$

where: Δs_{rj} - increament of permanent stresses calculated from the relation (3.1).

At the moment when on the area of a particular layer the mounting-up of tensile stress-waves generated by an impulse R_j takes place, there arises there simultaneously the permanent increment of tensile stresses due to the carry- over effect.

As it has been mentioned already the distributions of bond strength in particular layers are not indentical. Therfore the process of damage in each of the linked layers will develop differently in respective time.

Still it can be assumed approximately that the value of the permanent increase of stresses in both linked layers will be indentical, equal Δs_{rj}.

(2) The rightness of the hypothesis on linked layers has been confirmed by experimental results which prove that in brittle materials (like brittle steel, concrete), two planes of destruction can develop simultane-ously [2, 3]. According to Miklowitz the second plane of destruction takes effect owing to the stress-waves generated at the moment the destruction in the first plane take place [2]. According to the theory set up by the autor both planes of destruction can be regarded as linked layers subjected to the simultaneous destruction, emitting towards each other tensile stress-waves that mount up within the area of both layers.

Thus the maximum increment of stresses both permanent and temporary ones, attain the value

$$\Delta s_j \approx 3\,\Delta s_{rj}. \tag{3.4}$$

4. INTERFERENCE OF STRESS-WAVES GENERATED BY GROUP IMPULSES. CLASSIFICATION OF PROCESSES.

4.1 Moderate rate processes.

Until now, it has been assumed while investigating the processe of damage, that they proceed in conditions of a slow increase of loading. This assumption has the same meaning as the one which states that the stress-waves generated during each increment of the damage (consisting in the breaking of the bonds group class R_j) are being damped before the subsequent increment of damage R_k takes place. Consequently, neither the participation of those waves in the destruction, of bonds group R_k nor, their interference with the waves generated by impulse R_k, is possible.

Now let us consider the moderate rate processes, that is those in which the span of time ΔT contained between the moments of destruction the successive bond groups (R_j and R_k) is insufficient for total damping of stress-waves generated by impulse R_j before the impulse R_k appears. Consequently, in the moderate rate processes it comes to the interference of stress-waves generated by the successive increments of damage.

In order to facilitate the analysis of moderate rate processes we shall consider a model with a critical layer situated centrally.

In fig. 4.1 there has been presented a partial diagram showing the relation between time and the actual stresses, for the central contact layer. The diagram refers to the initial phase of the moderate rate processes.

In the time range corresponding to the development of damage, the resultant actual stresses represent the sum of:

a) Permanent stresses due to external loading / broken line /.
b) Increments of permanent stresses due to the damage / continuous line with jumps /.

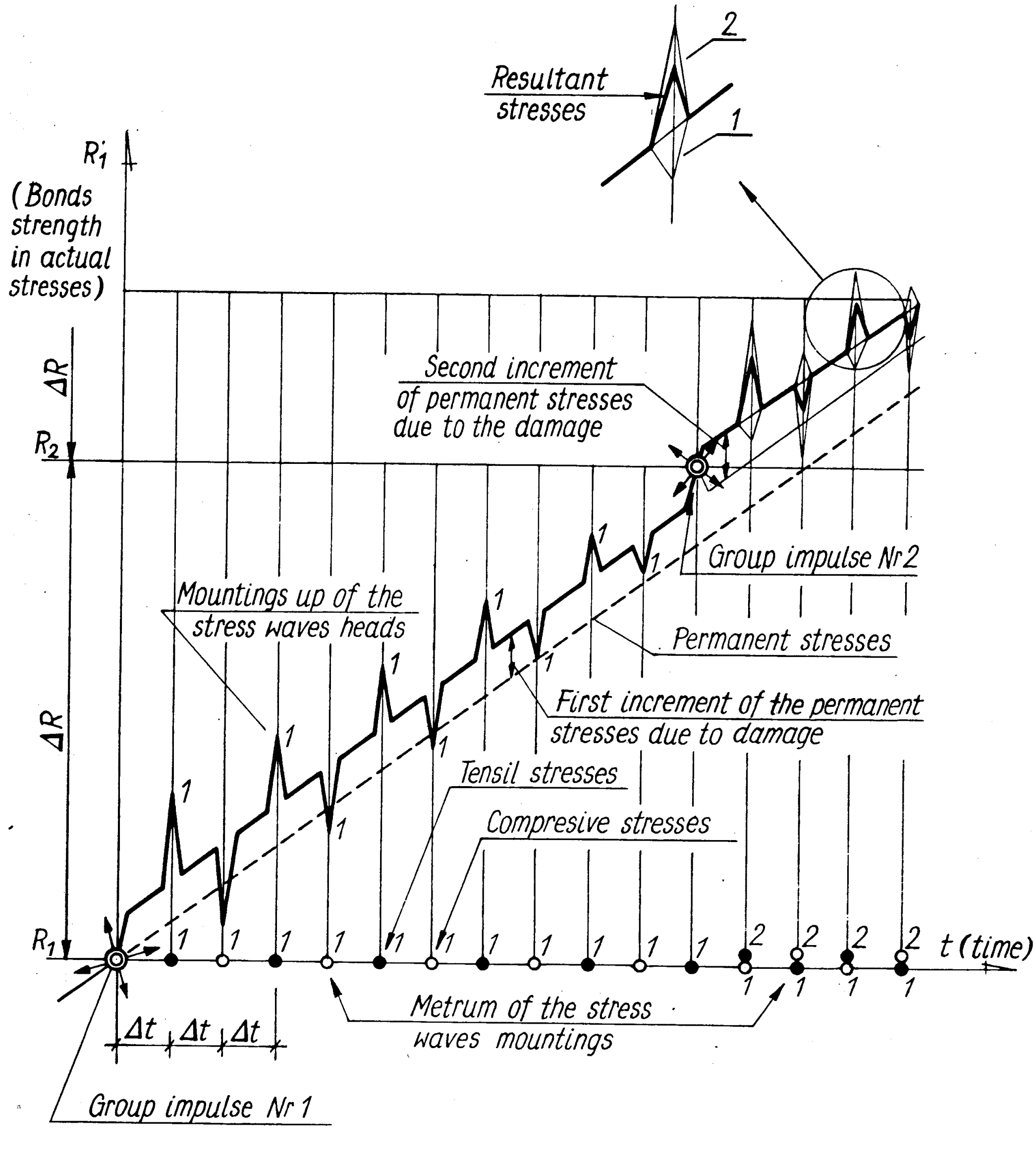

Fig. 4.1

c) Mounting-up of stress-waves heads / double triangles /; these stresses has been presented with the damping effect duly accounted for; consequently the heights of the triangles depicting the mounting-up of stresses, decrease in time.

On the graph have also been indicated the moments the group impulses arise (star marks), and the metrum of stress-wave mountings (circles). The metrum equals Δt, where Δt is the value obtained from the relation (3.2).

From fig. 4.1 it is evident that the breakage of the second in turn bond- group of a strength R_2 has taken place with the mounting-up of tensile stress- waves participating in it. These waves are generated by the group impulse that developed in the course of the breaking up of bonds of the strength R_1. In consequence of the above the metrum marked on the time axis by the points of mounting-up of the stress-waves generated by the second gruop impulse (R_2) have to coincide with the metrum of the mountings-up of stress-waves induced by the first impulse (R_1). Yet the mountings-up of stress-waves, generated by these successive group impulses (e.g. R_1 and R_2) are shifted in respect of each other by Δt and are therefore of the oposite sign. The interference of stress-waves is in these circumstances a destructive one. In a case of no damping it would result in the total mutual cancellation of all stress-waves, on assumption that the temporary stresses induced by the successive group impulses differ by sign only.

Actually however the effect of damping always exists in the moderate rate processes and the stress-wave initiated by an earlier impulse (R_1) before it attains the point of interference with a stress-waves induced by the later impulse (R_j) will be partially reduced. Consequently, in spite of the destructive interference of both waves, only the said partial reduction of the mounting up of the tensile stress-waves, generated by the later impulse R_j, is taking place and the time of these mountings in moderate rate processes will continue to be the time of breaking of the successive bond groups (belonging to the class of strength R_k, R_l ... and so on).

4.2. Coordination of the moderate rate processes. Comulation of the temporary stresses.

Implications arising from the latter chapter refer to the coordination in the process of damage. This consists in the simultaneousness of the moments of mountings of stress-waves induced by the successive group impulses while the destructive wave interference accompany them.

The above principle can be expressed by the following relation

$$\Delta T = m.\,\Delta t \tag{4.1}$$

where: ΔT - period of time between moments of destruction of two successive bond classes (R_i, R_j); it depends on the rate of increase in loading.

m - natural figure.

Δt - time of a full cycle of stress-wave in a model determined by the equation (3.2).

Further on, the foregoing processes shall be called the coordinated processes. In those processes the time span ΔT between moments of destruction of the successive bond groups is equal m-th multiple of time Δt (where Δt is a full cycle of stress-waves in a model). It has to be noted that in the individual phasis of the process the number m can change according to the local rate of stresses.

In the conditions described, s_r - the resultant temporary stress in the given layer of the model at any section at a given time, consists of the sum of all mountings of stress-waves generated since the beginning of the process i.e.

$$s_r = 2\sum_{i=1}^{i=r} \Delta s_i \ (-1)^i \ \varphi[(r-1)\,\Delta T] \tag{4.2}$$

where: i - ordinal number of a successive broken group (classes) of bonds contained within limits <i,r>.

ΔT - period of time elapsing between moments of destruction of two successive groups (classes) of bonds; it depends on the rate of the process.

Δs_i - increments of temporary stresses generated by successive group impulses (the value of those increments can by calculated from the formula (3.1).

$\varphi\,[(r-1)\,\Delta T]$ - damping function,

$(-1)^i$ - a factor accounting for the fact that the successive stress-waves change their sing (they are in turn tensile or, compressive stress-waves).

From the relation (4.2) it results that the lower the rate of the process (the longer time-periods ΔT) the higher the damping and the greater the resultant stress is.

In low rate processes all stress-waves, before falling into interference with waves generated by the last impulse (r) are practically subject to complete damping. In that case the problem of temporary stresses has been reduced to a mounting-up of stress-waves generated by the last impulse (r) and the value of temporary stresses obtained, is the greatest of all.

The lowest value of temporary stresses appears in the fast rate processes. The effect of damping can be then ignored. Owing to the destructive interference of stress-waves generated by the successive group impulses the temporary stresses disappear and then the effect of dynamic factor on the process of destruction is completely eliminated.

4.3. Very high rate processes.

An upper limit of the moderate rate processes can be determine by means of the ultimate rate stress

$$\dot{s}_{\lim} = \frac{\Delta R}{\Delta t} = \Delta R \frac{c}{L}. \tag{4.3}$$

In a case when the rate of stress attains a value higher then the ultimate one defined by the formula (4.3), then the "metre" of mountings of stress-waves set up by successive group impulses are due to be shifted in respect of each other (fig. 4.2) while the process loses its coordinated character. This sort of processes shall be called the very fast rate ones.

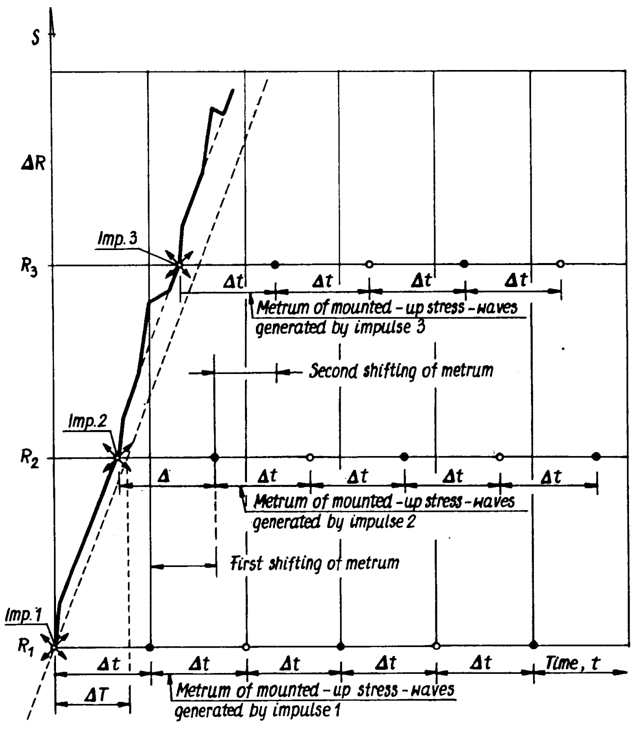

Fig. 4.2

In connection with non-coordinated procedure of the very high rate processes the destructive interference of stress-waves generated by the successive impulses, disappears. In consequence of that the return of the influence of the dynamic factor can be noted. It appears suddenly, immediately after the ultimate velocity rate of stresses (determined by the relation (4.3)) has been exceeded. It is featured by the mounting of tensile stress- waves which accompany the destruction of each of the bond groups belonging to the successive strength classes. Those mounting-ups are superposed on the increments of permanent stresses evolved by the carry-over effect that at the critical phase of the process has a negative influence on the strength of the model.

Thus, at the moment of exceeding the ultimate rate, the process characterised by the completely neutralized dynamics, changes rapidly into a process of the procedure entirely dynamic. At the same time a rapid fall in the strength of the model has to be expected as that has been observed at some experimental investigations [11].

However, in some of the very high rate processes, in spite of the influence of the dynamic factor, a considerable growth in the tensile strength takes place [3]. A causative interpretation of this phenomenon lies beyond the scope of the possibilities offered by the proposed brittle material model which is based on the principles of classical mechanics. It requires a reference to the methods and theories used by the physics of the solids [11].

5. INFLUENCE OF THE NEUTRALIZING ACTIVITY.

The physical feature of the neutralizing activity consists in an elimination from the process the influence of the temporary tensile stresses.

So far the author has disclosed basically the existence of three mechanisms of the neutralizing activity, namely:

a) Effects linked with the rate of loading application, consisting in the destructive interference of stress-waves in the coordinated processes (It has been dealt with in the preceding chapter).
b) Effects of the gradient of the area of deformations, the mechanism of which consists in the annihilation of the simultaneousness of the group impulses in the whole area of stresses (The said simultaneousness exists in the homogeneous deformity area).
c) Effect of the rigid compensating system of the type used by Balawadze [5], Evans and Marathe [4] and the author himself in the experimental research on the concrete in tension [6].

At this point we shall limit the scope of work to the description of principles governing the functioning of the former of the above mechanisms.

An outline of the compensating device is presented in paper [1] p. 86 fig. 4. ("Arrangement of the compensatory-neutralising system").

It shows a rigid steel structure and a sample of a brittle material fixed inside. This arrangement ensures that the external loading in transmitted to the sample by means of the above device.

The neutralizing action of the system consists on the fact that the waves of the compressive stresses generated at the edges of the critical layer, falling on the surfaces of end plates are reflected by them, without undergoing then an alteration of phase (that would have taken place in a case of a model subjected to deformation without the compensating system). The reflected waves remain therefore the compressive stress-waves. In the described conditions there is no possibility for the temporary tensile stresses in the model to mount up. The nature of the mechanism of the neutralizing activity consists therefore in this case in preventing the reversal of the phase of stress-waves at their reflection from the end faces of the model to take place.

Apart from the compensatory-neutralizing systems there can be met in the laboratory practice the compensating systems which do not posses the neutralizing properties. Those systems transmitted to the mechanical devices of specially braced structures strength testing machinery are characteristic for their low deformation adaptability and accordingly their ability to cumulate elastic energy is practically impossible. It is possible therefore, at a very low rate of the compulsory deformations of the sample to continue the process of loading within the range of the falling portion of the graph representing the relation σ - ε. The above systems has been applied, among others, by Hughes and Chapman in thus tests on concrete in tension [7].

6. MECHANISM OF FRACTURE FOR THE PROCESSES OF A DYNAMIC PROCEDURE.

6.1. Precesses of dynamic procedure.

Conformely to the chapter 4 as processes of dynamic procedure we shall consider the low rate processes and the very high rate processes.

In the first, owing to the damping, the destructive interference of stress waves generated by the successive impulses does not occur, while in the second, the same result is reached owing to the shifting of metrum of the mounting-up of stress waves generated by the consecutive impulses.

In case of the processes with the fully dynamic procedure the problem can be reduced to an analysis of singular group impulse. The value of the temporary stresses is then calculated on the base of formula (3.1) with application of the coeficient 2. In this chapter the low rate processes are analised.

At the initial stage of slow rate processes the stress-waves generated by group impulses do not exert any influence on the destruction process and their energy becomes dispersed in the damped vibrations. This situation changes at the suitably advanced stage of the process, when the temporary stresses attain a considerable value. Their influence give rise to the detached spontaneous effects, preceding the avalanche stage of the process.

6.2. Semi-spontaneous phase of the process.

So has been named that part of the process at which detached spontaneous effects appear but the destruction process develops in the conditions of increasing loadings. This phase is located in the raising section of the relation σ - ε and approaches the boundary of the falling portion of the said relation, corresponding to the phase at which the destruction process proceeds entirely spontaneously i.e. avalanche-like.

The first spontaneous effect of the first order consists in the fact that due to an increment of permanent and temporary stresses which follows the breaking of bonds class $R_{r/1}$, there takes place in the critical contact layer the break of bonds belonging to the next class of strength (i.e. bonds of the class $R_{r/1+1}$) without an increment of the outer load.

Therefore the criterion for the first spontaneous effect of the first order takes the following form

$$\Delta s_{I/1} = \Delta R \tag{6.1}$$

where: $\Delta s_{I/1}$ - an increment of the temporary and permanent stresses, resulting from the breakage of bonds belonging to the class $R_{r/1}$ of strength, the value of which is calculated from relations (3.1) and (3.4).

$\Delta R =$ const - an increment covering the difference between strength of bonds belonging to two successive classes of strength.

Making use of the relations (3.1) and (3.4) and carrying out the required substitutions and modifications, we obtain

$$3\,[\varphi(R)]_{R=R_{I/1}}\;R_{I/1} = \int_{R_{I/1}}^{R_{max}} \varphi(R)\,dR\,. \tag{6.2}$$

The unknown quantity which appears in relation (6.2) in an entangled form is the actual stress $R_{r/1}$, equal the strength of bonds after the braking of which the first spontaneous effect in the critical contact layer develops.

From the moment the first spontaneous effect of the first order takes place, the next of the same order will develop in the result of growing of outer loading (i.e. the second spontaneous effect of the first order, the third one, and so on).

The mechanism of the growth of damage at this stage of process is as follows.

After the break of bonds group due to an increment in outer loading the stress- waves give rise to a spontaneous effect and immediately enter into destructive interference with the stress-waves produced by that effect.

Both groups of bonds i.e. the one that by its breakage a spontaneous effect is evolved and the other that has been destroyed spontaneously, are situated directly in the vicinity on the strength axis and the moments of their destruction are separated by very short time spans (Δt). It signifies that the absolute values of temporary stresses produced by the destruction of each of the two bonds groups shall be nearly identical and therefore the stresses must balance each other.

Therefore after each spontaneous effect has made its appearance the destruction of the next bonds group can take place only on condition of further increment in the outer loading.

Thus, at a phase of the process corresponding to the period of development of spontaneous effects of the first order, the increments of damage are due in a half to the dynamics of the process and in a half to the increments of outer loading.

At the duly advanced stages of the semi-spontaneous phase there appear spontaneous effects of higher orders.

The first spontaneous effect of the second order consists in the fact that under the influence of the increment of permanent and temporary stresses caused by the breakage of bonds class $R_{II/1}$ these comes to a spontaneous break of bonds belonging to two successive classes of strength (i.e. bonds class $R_{II/2}$ and $R_{II/3}$).

Consequently the first spontaneous effect of be third order consists in the fact that under the influence of the increment of permanent and temporary stresses caused by the breakage of bonds class $R_{III/1}$ there comes to a spontaneous breakage of bonds belonging to three successive classes of strength (i.e. bonds class $R_{III/2}$, $R_{III/3}$, $R_{III/4}$) etc.

Between moments of arising of the first spontaneous effects belonging to the successive orders there are going to develop successive spontaneous effects of those orders i.e. the second spontaneous effect of the second order the third spontaneous effect of the second order the second spontaneous effect of the third order, the third spontaneous effect of the third order etc.

At the phase of the process corresponding to the period of spontaneous effect of the second order arising every third bonds group is subjected to destruction owing to an increment in outer loading while the remaining bonds groups are bound to be destroyed owing to the dynamics of the process. Similary at the phase corresponding to the period of spontaneous effects of the third order every forth bonds groups is subject to breaking owing to an increament in outer loading while the remaining bonds group (three groups of bond strength) are bound to be destroyed owing to the dynamics of the process.

6.3. Evaluation of spontaneous effects of the higher orders.

A criterion difining spontaneous effects of the higher orders has been formulated in the same manner as in the case of the first spontaneous effect while the essentials of the definitions applied to those effects in preceding chapter have to be taken into account.

Thus in case of the first spontaneous effect of the second order that criterion (in accordance with the description) takes the following formula:

$$\Delta s_{II/1} = 2\Delta R \tag{6.3}$$

where: $\Delta s_{II/1}$ - the maximum increment of temporary and permanent stresses tied up with breaking of bond class $R_{II/1}$ (which produces the first spontaneous effect of the second order).

The coefficient 2 appearing on the right-hand side of the formula (6.3) expresses the fact that the spontaneous effect of the second order consists in the simultaneous destruction of bonds belonging to two successive strength classes (consequently of the moment that effect has been achieved, the increment on the axis of actual stresses, equals $2\,\Delta R$).

Modifying the left-hand side of the formula (6.3) we obtain:

$$\Delta s_{II/1} = \frac{3\,[\varphi(R)]_{R=R_{II/1}}\ \Delta R \,.\, R_{II/1}}{\int_{R_{I/1}}^{R_{max}} \varphi(R)\,dR - 2\int_{R_{I/1}}^{R_{II/2}} \varphi(R)\,dR} \,. \tag{6.4}$$

Similarly as in the case of the definition of the first spontaneous effect of the first order the numerator of the formula (6.4) presents the sum of forces that have been carried by the broken bonds class $R_{II/1}$, multiplied by dynamic coefficient; the denominator presents the area of the critical contact layer with partial damage duly accounted for.

The first term appearing in the denominator of the formula (6.4) presents the area of the damaged contact layer after arising of the first spontaneous effect of the first order. The second one presents the increment of damage produced during the period elapsed between the first spontaneous effect of the first order and the first spontaneous effect of the second order. In that period each increment of damage obtained under the effect of growing loading is always accompanied by only one spontaneous effect expressed by breaking of bonds belonging to the next in turn strength class. That has been taken into account by introducing a multiplier 2.

By substituting formula (6.4) into the relation (6.3) and by suitable modifications we obtain the final formula difining the first spontaneous effect of the second order as follows

$$\frac{3}{2}\,[\varphi(R)]_{R=R_{II/1}} R_{II/1} = \int_{R_{I/1}}^{R_{max}} \varphi(R)\,dR - 2\int_{R_{I/1}}^{R_{II/1}} \varphi(R)\,dR \,. \tag{6.5}$$

Applying the same method we obtain a formula defining the process of development of the first spontaneus effect of the third order in the following form:

$$\Delta s_{III/1} = 3\,\Delta R \tag{6.6}$$

where

$$\Delta s_{III/1} = \frac{3\,[\varphi(R)]_{R=R_{III/1}}\,\Delta R\,R_{III/1}}{\int_{R_{I/1}}^{R_{max}} \varphi(R)\,dR - 2\int_{R_{I/1}}^{R_{II/1}} \varphi(R)\,dR - 3\int_{R_{II/1}}^{R_{III/1}} \varphi(R)\,dR}. \tag{6.7}$$

Thus the definition expressing the first spontaneous effect of the third order takes the form

$$\frac{3}{3}\,[\varphi(R)]_{R=R_{III/1}}\,R_{III/1} = \int_{R_{I/1}}^{R_{max}} \varphi(R)\,dR - 2\int_{R_{I/1}}^{R_{II/1}} \varphi(R)\,dR - 3\int_{R_{II/1}}^{R_{III/1}} \varphi(R)\,dR. \tag{6.8}$$

While the criterion for the first spontaneous effect of the random m-order can be expressed in the general form

$$[\varphi(R)]_{R=R_{m/1}}\,R_{m/1} = \tag{6.9}$$

$$= \frac{m}{3}\left[\int_{R_{I/1}}^{R_{max}} \varphi(R)\,dR - 2\int_{R_{I/1}}^{R_{II/1}} \varphi(R)\,dR - 3\int_{R_{II/1}}^{R_{III/1}} \varphi(R)\,dR - \ldots - m\int_{R_{m-1/1}}^{R_{m/1}} \varphi(R)\,dR\right].$$

The relation (6.9) can take also the form:

$$[\varphi(R)]_{R=R_{m/1}}\,R_{m/1} = \frac{m}{3}\left[\int_{R_{I/1}}^{R_{max}} \varphi(R)\,dR - \sum_{n=1}^{n=m} n \int_{R_{n-1/1}}^{R_{n/1}} \varphi(R)\,dR\right]. \tag{6.10}$$

Naturally the quantity n appearing in the formula (6.9) and (6.10) represents the order of the successive spontaneous effect.

The value n varies from 2 to m.

6.4. The state of the ultimate bearing capacity of the model. Criterion of strength for the processes of dynamic procedure.

On the ground of the relation (6.1) - (6.10) we can calculate the values of bond strength $R_{1/1}$, $R_{II/1} \ldots R_{n/1}$

Their destruction releases spontaneous effects of the successive orders and enables us to calculate the values of corresponding nominal stresses by applying the following formulas:

$$\sigma_{I/1} = R_{I/1} \int_{R_{I/1}}^{R_{max}} \varphi(R)\, dR \tag{6.11}$$

$$\sigma_{II/1} = R_{II/1} \left[\int_{R_{I/1}}^{R_{max}} \varphi(R)\, dR - 2 \int_{R_{I/1}}^{R_{II/1}} \varphi(R)\, dR \right]$$

$$\sigma_{III/1} = R_{III/1} \left[\int_{R_{I/1}}^{R_{max}} \varphi(R)\, dR - 2 \int_{R_{I/1}}^{R_{II/1}} \varphi(R)\, dR - 3 \int_{R_{II/1}}^{R_{III/1}} \varphi(R)\, dR \right]$$

....

....

$$\sigma_{m/1} = R_{m/1} \left[\int_{R_{I/1}}^{R_{max}} \varphi(R)\, dR - \sum_{n=2}^{n=m} n \int_{R_{n-1/1}}^{R_{n/1}} \varphi(R)\, dR \right] .$$

It has to be noted that the characteristic feature of the processes of dynamic procedure is the considerably larger range of damage than at the processes of the neutralised procedure.

This fact reflects in the relation (6.11) where the comparatively low actual stresses are accompanied by large damage. That is the effect of the dynamics of the process.

The maximum valus of the stress $\sigma_{m/1}$, σ_{rD}, ascertained by the relation (6.11) corresponds to the ultimate bearing capacity of the model.

Relation (6.10) and (6.11) together presents the required new criterium of strength which takes into account the effect of dynamics in the process of fracture.

Farther on, the stress σ_{rD} and corresponding actual stress R_{rD} shall be called "the dynamic strength" of the model.

7. CRITERION OF STRENGTH IN PROCESSES OF FULLY NEUTRALIZED DYNAMICS.

In accordance with the text of chapter 5, the process with fully neutralized dynamics is featured by the elimination of the effect of temporary stresses. This means that coefficient of stresses mounting up, receives the value 1 (instead of 3, as in the process with non-neutralized dynamics).

In the above conditions the appearance of the first spontaneous effect is of the same meaning as the achievment of the ultimate bearing capacity of the model, i.e. of the avalanche phase.

Criterion of strength for the process with neutralized dynamics will take the form

$$\Delta R = \frac{[\varphi(R)]_{R=R_{rN}} \, \Delta R \; R_{rN}}{\int_{R_{rN}}^{R_{max}} \varphi(R) \, dR} . \tag{7.1}$$

After modifications, the final form of the criterion is:

$$[\varphi(R)]_{R=R_{rN}} \; R_{rN} = \int_{R_{rN}}^{R_{max}} \varphi(R) \, dR . \tag{7.2}$$

In publications dealing with the statistical theory of strength relation (7.2) is brought up along the different line of analysis than the relations developed by the author.

8. COMPARISON OF THE PROCESSES WITH NON-NEUTRALIZED AND NEUTRALIZED DYNAMICS.

8.1. Assumptions to the analysis.

The comparative analysis deals with the cases of strength presented in actual stresses, also in nominal stresses, as well as the ultimate elongations and the densities of the energy of fracture.

Considering that all criteria possess an entangled form, the above analysis can be, practically, carried out only by the method of comparing the results of calculations applied to a case of a chosen distribution of strength of the critical layer bonds. For this purpose we shall use the possibly simplest distribution, namely the constant one, expressed by the relation

$$\varphi(R) = \frac{1}{R_{max}} \tag{8.1}$$

which is determined within the range <$0, R_{max}$>.

8.2. Processes of the dynamic procedure.

Substituting function (8.1) into the relation (6.2) which defines the first spontaneous effect of the first order, we obtain the following formula:

$$3\frac{R_{I/1}}{R_{max}} = \int_{R_{I/1}}^{R_{max}} \frac{dR}{R_{max}}. \tag{8.2}$$

Hence, the strength of bonds after the breaking of which the said spontaneous effect appears,

$$R_{I/1} = 0{,}25\, R_{max}\,. \tag{8.3}$$

At the moment the first spontaneous effect of the first order takes place, in the critical layer of the model the nominal stresses are existing

$$\sigma_{I/1} = 0{,}25 R_{max} \left(1 - \int_{0}^{0{,}25R_{max}} \frac{dR}{R_{max}}\right) = 0{,}1875 R_{max}\,. \tag{8.4}$$

At the same stage of the process, the deformation of the critical layer equals

$$\varepsilon_{I/1} = \frac{R_{I/1}}{E} = \frac{0{,}25\, R_{max}}{R} = 0{,}25\ \ \varepsilon_{max}\,. \tag{8.5}$$

Now the spontaneous effects of the higher order can be determined by applying for this purpose relation (6.10) and (6.11). The results obtained are set up in table I and plotted on the diagram representing relation σ - ε (fig. 8.1) in which a falling portion has to be noted. In the experimental conditions this portion can be obtained at conditions after compensaiting systems have been applied (see chapter 5).

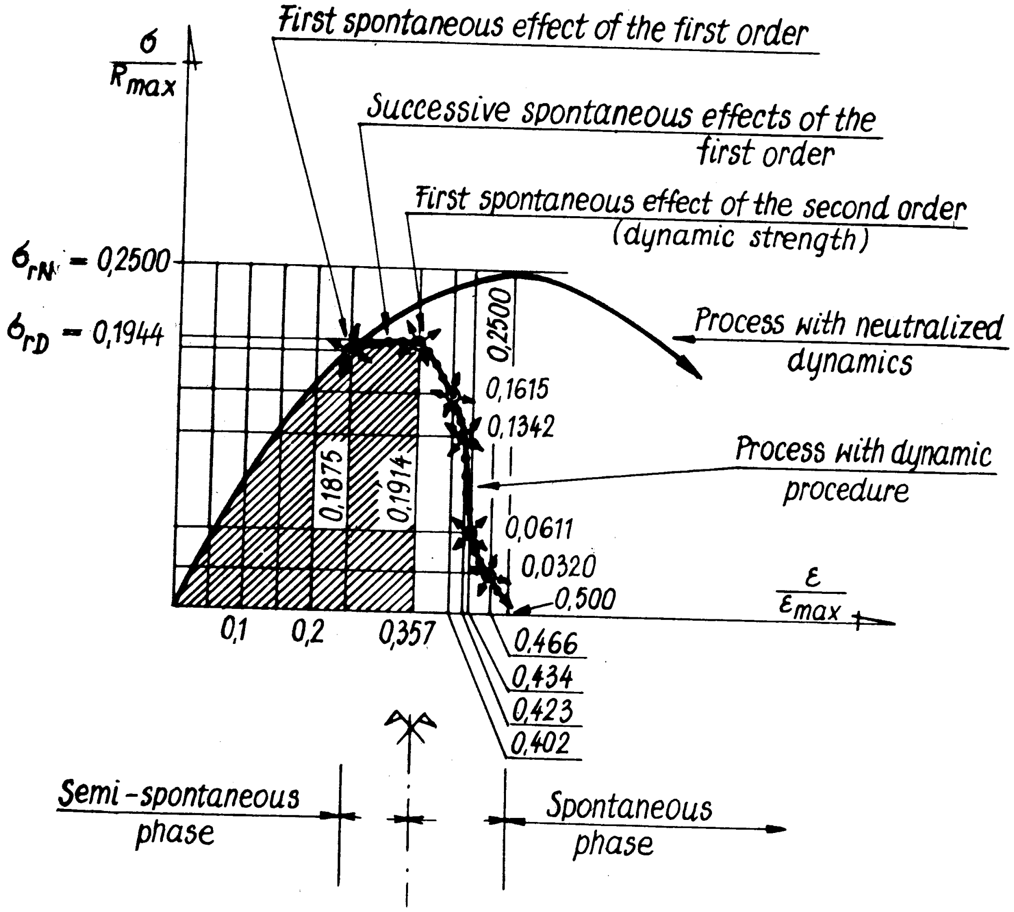

Fig. 8.1

TABLE 1.

Results of analysis concerning spontaneous effects of the orders I to VI

Order of the effect	I	II	III	IV	V	VI
Strength of bonds $\frac{R_n}{R_{max}}$	0,250	0,357	0,4016	0,4229	0,4312	0,466
Nominal stresses $\frac{\sigma_n}{R_{max}}$	0,1875	0,1944 σ_{rD}	0,1615	0,1342	0,0611	0,0320

The analysis carried out proves that in the conditions of a process with a dynamic procedure, the ultimate bearing capacity is reached at the moment of appearing of the first spontaneous effect of the second order. The actual stresses attain then the value

$R_{II/I} = 0{,}357 R_{max}$, the strains

$\varepsilon_{II/I} = 0{,}357 \frac{R_{max}}{E} = 0{,}357\, \varepsilon_{max}$, and the nominal ultimate stresses,

$\sigma_{II/I} = \sigma_{rD} = 0{,}1944 R_{max}$.

8.3. Process with neutralized dynamics.

We substitute function (8.1) into the formula (7.2) and obtain the relation:

$$\frac{R_{rN}}{R_{max}} = \int_{R_{rN}}^{R_{max}} \frac{dR}{R_{max}} . \tag{8.6}$$

From this relation we can find that $R_{rN} = 0{,}5\ R_{max}$. The strength of the model expressed in nominal stresses $\sigma_{rN} = 0{,}25\ R_{max}$, and the ultimate strain for the critical contact layer, $\varepsilon_{rN} = 0{,}5\ \varepsilon_{max}$.

We shall calculate now the densities of the fracture energy. In order to do it we express the nominal stresses as a function of strain

$$\sigma(\varepsilon) = \varepsilon E \left(1 - \frac{\varepsilon}{\varepsilon_{max}}\right). \tag{8.7}$$

The density of energy at the strain ε_i, is expressed by the relation

$$V_r = \frac{1}{2} E \int_0^{\varepsilon_i} \varepsilon^2 \left(1 - \frac{\varepsilon}{\varepsilon_{max}}\right) d\varepsilon = \frac{1}{2} E \left(\frac{\varepsilon^3}{3} - \frac{\varepsilon^4}{4\varepsilon_{max}}\right)\Bigg|_0^{\varepsilon_i} . \tag{8.8}$$

Thus it is obvious that in the case of the effect of damage being taken into account, the density of the energy is represented by a function of the third order of strains.

In a case of a process with neutralized dynamics the ultimate strain displacement equals 0,5 ε_{max} and there-fore the density of the energy of destruction, $v_r = 0{,}0130\, E\, \varepsilon^3_{max}$.

Now, considering the relation (8.3) and assuming the strain to equal 0,357 ε_{max} we can evaluate the density of the energy of fracture in a process of the dynamic procedure. This density is $v_r = 0{,}00555\ E\ \varepsilon^3_{max}$.

In the last case the estimation is only an approximate one since from the moment of the first spontaneous effect place, the graph representing relation σ - ε for a process with the dynamic procedure changes its shape. The actual energy of destruction is therefore slightly smaller.

The results of the analysis carried out are arranged in Table 2 (and in fig. 8.1). The data presented there allow to draw a conclusion that all values referring to a process with the neutralized dynamics are much more advantageous than those for a process with the dynamic procedure.

TABLE 2.
Comparison of the processes

Index name / Type of Process	Strenght R_r /Actual stresses/	Strenght σ_r /Nominal stresses/	Strain ε_r at the moment of attaining the ultimate bearing capacity	Density of the fracture energy V_r
Process with the neutralized dynamics	0,50 R_{max}	0,25 R_{max}	0,50 ε_{max}	0,0130$E\,\varepsilon_{max}^3$
Process with the dynamic procedure	0,357 R_{max}	0,1914 R_{max}	0,357 ε_{max}	0,00555$E\,\varepsilon_{max}^3$
Relation: A neutralized process to a non-neutralized process	1,4	1,30	1,4	2,35

$$\varepsilon_{max} = \frac{R_{max}}{E}$$

9. DYNAMICS OF THE PROCESS IN THE ASPECT OF EXPERIMENTAL RESEARCH ON CONCRETE IN TENSION.

9.1. Low rate processes with neutralized dynamics.

Evans and Marathe obtained a falling part of the σ - ε relation for concrete in tension [4]. There has been applied a compensatory-neutralizing contrivance similar to the type of system described in chapter 5.

It has been stated that the falling part of the relation σ - ε can be obtained on condition of maintaining a suitable rate of strain which equale 25 μs/min. This value can be regarded as the "rate of relaxation".

In the aspect of the problem considered the most significant result of this experiment are observations concerning the rising part of the graphs σ - ε. They indicate that either the value of the maximum nominal stresses or accompanied them strain obtained in the compensatory-neutralizing system, are considerably greater then those of ultimate strains and stresses established in the course of the tests on axial tension, usually carried out.

Similar conclusions can be drawn from the investigations carried out by Balawadze [5] who, in his experiments, applied compensatory-neutralizing systems based in their action on a principle similar to that applied in system used by Evans and Marathe.

The facts described above can be explained as the results of the neutralizing effect of the applied systems which prevented the tensile stress-waves to arise.

Here we duly recall the results of the measurements of the surface energy carried out in connection with the adoption of the crack-theory to concrete. It has been proved that the value of that energy depends on the method of tests. E.g., by applying the "analitical" method the experimental base of which is a test on axial tension and than the method "slow bending", the base of which is bending, a number of researchers that we qoute, after Radjy and Hansen [8], have stated the following: the value of surface energy calculated on the base of bending test are several times greater than those determined by the test on axial tension. The researchers cited above made a commentary explanining that the results obtained were due to the difficulty in achieving sufficiently slow progress in destruction during the axial tensile test. In other words the higher results obtained in slow-bending tests can be interpreted as an effect of neutralizing activity of the gradient in the area of strain on the dynamics of the process.

9.2. Tensile strength of concrete in function of the strain rate.

The experimental dependence concerning the subject of the strain rate effect on tensile strength of concrete is presented in figure 9.1. When ploting this diagram we quote the research carried by Heilmann, Hilsdorff and Finsterwalden [9], Nicolau [10] and Zielinski [3].

The rates of moderate rate processes are higher than the "relaxation rates" fixed in tests [4], hence a conclusion could be drawn, that an increase in strength observed at the gradual passing to the higher rate processes is not due to the elimination of sliding mechanisms but due

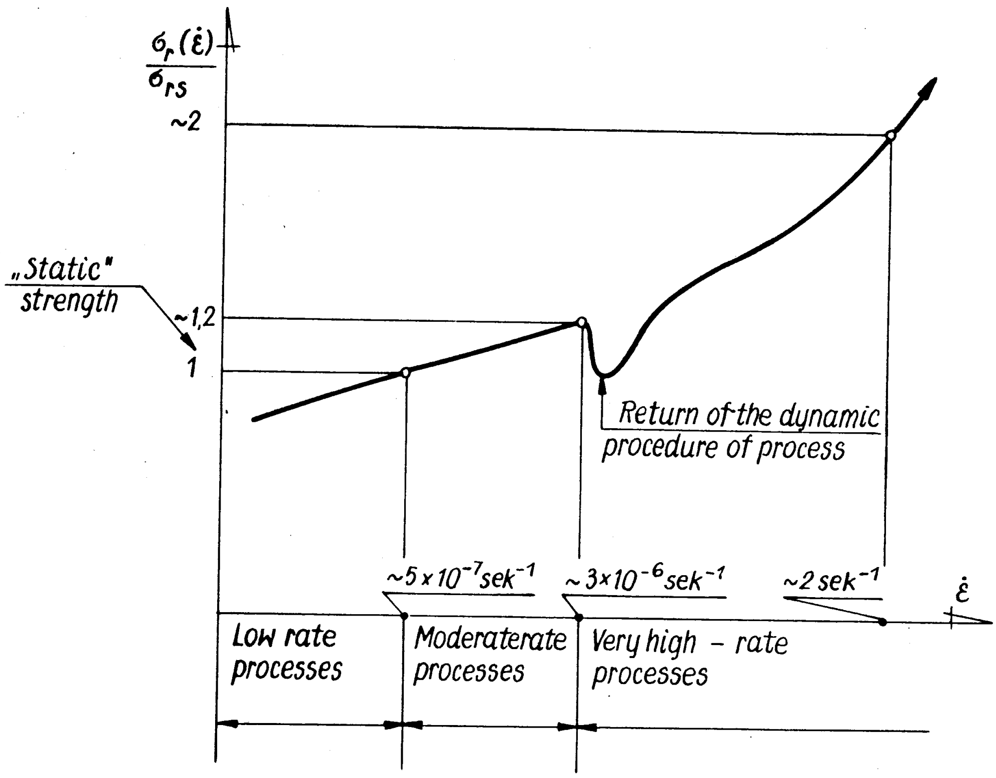

Fig. 9.1

mainly to the neutralisation of the effect of dynamics of the process taking place in the form of a destructive interference of stress-waves induced by the successive group impulses.

Increasing the strain rate, until the rate of about 10^{-1} sec^{-1} is attained, the strength rapidly decreases failling down to the value below static strength (obtained at the low rate process).

According to the opinion of the author this rapid fall in strength (noticed by Nicolau [10]) should be interpreted as the result of attaining the limit rate of the moderate rate processes with the recurrence of the influence of the dynamics of the process (what is in coformity with the relation (4.3)).

After exceeding the limit rate of the moderate rate process a considerable increase of tensile strength take place. A causative explanation of this phenomenon lies beyound the scope of the classic mechanics (on which the model of the brittle material analysed in this paper is based) and requires reference to notions and terms used by the physics of solides.

10. CONCLUSIONS.

From the analysis of the presented model of the brittle material can be inferred that, the dynamic factor plays an essential part in the mechanism of fracture.

The mechanism of its operation consists in the forming of mounting up of the stress-waves in the area of the critical layer, where they are generated in the course of the successive increments of damage which develop by abrupt rises.

After being reflected from the end surface of the sample the stress-waves return back in the form of tensile stress-waves on to the area of the critical layer of the material. There it comes to their superposing on the permament stresses due to the external load. In this way the energy evolved in course of the damage process substitutes partially the external forces in their task of destruction of material.

The effect of the dynamic factor can be eliminated by the application of neutralizing factors.

The strength, the ultimate deformation, the energy of fracture achieved at the processes with neutralized dynamics procedure attain the value higher than those at the processes with non-neutralized dynamics.

The justification of the presented hypothesis on mechanisms of brittle destruction seems to be confirmed by the facts observed in the course of experimental researches on concrete in tension.

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