

On the Influence of Cognitive Theories in the Teaching of Calculus in Austrian Secondary Schools at the Beginning of the 20th Century

LUCIANA ZUCCHERI AND VERENA ZUDINI (*)

Dedicated to the memory of Fabio Rossi

SUMMARY. - *In Austria, in the first decade of the 20th century, a reform in school curricula gave the opportunity to introduce the first elements of differential and integral calculus in the secondary school teaching. In some Italian speaking secondary schools of Trieste, a city which at that time belonged to the Habsburg Empire, these subjects were introduced in the teaching programmes. At the end of the First World War Trieste was annexed to the Kingdom of Italy. In our research we examine an example of the didactic methodologies used in Trieste before the First World War to teach these subjects and describe briefly the situation which occurred there in the period of transition from the regulations of the Habsburg Empire to those of the Kingdom of Italy.*

(*) This paper originated from a contribution of the authors to Working Group 1 of *ICMI Symposium Rome 2008: "The First Century of the International Commission on Mathematical Instruction (1908-2008) — Reflecting and Shaping the World of Mathematics Education"*.

Research approved by CIRD — Centro Interdipartimentale Ricerca Didattica of the University of Trieste and partially supported by Fondazione CRTrieste.

Authors' addresses: Luciana Zuccheri, DMI, University of Trieste, via A. Valerio 12/1, 34127 Trieste, Italy; E-mail: zuccheri@units.it

Verena Zudini, Department of Psychology, University of Milano-Bicocca, via Bicocca degli Arcimboldi, 8, 20126 Milano, Italy; E-mail: verena.zudini@unimib.it

Keywords: Mathematics Education, History of Mathematics Education, Secondary School, Calculus.

AMS Subject Classification: 97-03.

1. Introduction

The introduction of differential and integral calculus in the secondary school at the beginning of 20th century has been examined in the EM-ICMI Symposium “One Hundred Years of *L’Enseignement Mathématique*” (Geneva, October 2000), in which has been stressed the important role played by the journal *L’Enseignement Mathématique* (see Furinghetti [4]) and by the first *International Commission for Mathematics Instruction* (IMUK) (see Schubring [14]) in creating international communication and cooperation in mathematics education. In particular, an exhaustive description of the situation of the teaching of calculus at that time is contained in Kahane [10], which focuses on the report presented by the Hungarian mathematician E. Beke at the international conference of IMUK held in Paris in April 1914 (see Beke [2]), and deep reflections from the cognitive point of view, about the teaching of calculus, in order to get useful indications from the past, are expressed in Artigue [1].

Regarding the teaching of calculus in secondary schools, the situation in the first decades of 20th century was very different in the various countries. In particular, these subjects had been introduced in Austria, starting from 1909, in the teaching plans for the main types of secondary schools, whereas in Italy they were introduced in 1914 only in a type of school (*Liceo Moderno*) instituted in 1911. The situation of mathematics teaching in Italy and, in particular, the debate on the opportunity to introduce calculus in secondary teaching are widely described in Giacardi [6].

The case of the city of Trieste (Italy) and of the surrounding region Venezia Giulia, which at the end of the First World War were annexed to the Kingdom of Italy after having been for a long time part of the Habsburg Empire (in Austria), with the consequent institutional changes including the adaptation of the school system (regulations and teaching programmes) to that of the Kingdom of Italy, emphasizes the differences between the mathematics teaching in Austria and in Italy at that time. These differences were in the content, but mostly in the teaching methods, which in Italy were generally oriented to a theoretical and rigorous treatment of the mathematical subjects, whereas in Austria they were more oriented to practice and applications and took account of the indications of

IMUK and of the innovation programme of its president Felix Klein (see Zuccheri and Zudini [19], [20]).

In particular, during the examination of documents of that time concerning the teaching of mathematics in the city of Trieste (i.e. schools year-books, *Fondo Mathesis*), a teaching method, based on cognitive theories attributed to Ernst Mach, which we explain in the following, emerges. This method was esteemed by a group of mathematics teachers of the secondary schools of Trieste, Italian native speakers, which had a relevant role, from 1918 to 1923, in the process of adapting the mathematics teaching in Venezia Giulia secondary schools to that of the Kingdom of Italy. In Trieste, before the First World War, public secondary schools both in Italian and in German language were instituted; nevertheless, to be familiar with the German language and culture was essential for academic studies and certain professions, such as teacher in a secondary school. That was indeed because there were no Italian universities in the Habsburg Empire at that time, and, in fact, Graz and Vienna were usual choices of university for students from Trieste. This fact explains the openness of the Italian speaking mathematics teachers of Trieste to the Central European literature and philosophical thought.

2. The introduction of the first elements of differential and integral calculus in Austrian secondary schools

In Austria, a reform in school curricula which came into effect in the 1909/1910 school year¹ (performed by Minister Gustav Marchet) introduced the first elements of differential and integral calculus in the secondary school teaching, in the *Gymnasium*, *Realgymnasium* and *Realschule*. The school plans were very similar for the three school types. In particular, they prescribed the teaching of differential calculus in the *Gymnasium* and *Realgymnasium*, and that of differential and integral calculus in the *Realschule*.

The reform arose from the need to take into account the progress of science but also to modify the conception of teaching and its

¹Ministerial rules: no. 11662 of March 20, 1909, for the *Gymnasium*; no. 4620 of April 21, 1909, for the *Realgymnasium*; no. 14741 of April 8, 1909, for the *Realschule*.

methods. Regarding mathematics teaching, among the principles which inspired this reform there were the following: an adjustment to the pupils' grade of intellectual development; the simplification of courses through a stricter contact among the different branches, especially between arithmetic and geometry; a complete adjustment of the mathematical study to corresponding disciplines and to everyday life; the assimilation of the concept of function by taking any opportunity for developing it, until the study of the range of a function by means of the difference quotient. The plans were related to Felix Klein's programme in order to improve the teaching of mathematics.

The situation in the Italian speaking schools of Trieste was the following. Before 1909, in the *Gymnasium* and in the *Realschule* differential and integral calculus was not treated, though some problems about the determination of the tangents to a conic were introduced with algebraic arguments in the last years of course, by means of analytical geometry. In the 1909/1910 school year a section of *Realgymnasium* was instituted in the *Gymnasium*; this section moved to another Italian speaking *Gymnasium* instituted in 1912. In the "older" *Gymnasium*, starting from the 1909/1910 school year, the reform of the mathematics programmes was applied more gradually, first of all emphasizing the concept of function, but differential calculus was not introduced. On the contrary, in the Italian speaking *Realschule* the new instruction plans for mathematics were immediately applied since the 1909/1910 school year, giving very importance to the concept of functional dependence and introducing in the last year the concepts of differential and integral into the simplest applications in the subjects of mathematics and physics already treated. Also in the "new" *Gymnasium* (with the *Realgymnasium* section) the new teaching programmes were introduced closely, and immediately before the First World War they included:

- in the penultimate year of course, the "research of the slope [of a straight line] by means of the difference quotient";
- in the last year, "differential calculus, maxima and minima".

Which methods were used in teaching these subjects? The textbooks indicated in the respective school year-books before the end of

the First World War do not contain differential and integral calculus, either because these textbooks are edited before 1900 (Wallentin [17] and Močnik [12]) or because they treat only the subject matter for the first years (Jacob [7], [9]; Wallentin [16]). Clearly, the pupils studied these subjects on the notes taken during the mathematics lessons.

In a revised edition (of 1917) of Močnik's textbook for the last 4 years of the *Gymnasium* and *Realgymnasium* (Močnik and Zahradníček [13], not officially adopted in the considered schools), differential and integral calculus for one variable is treated without a previous rigorous introduction of the concept of limit of a function, and this way to consider the subject was approved by the school authorities. You can reasonably assume that the same happened also in the Italian speaking schools of Trieste. It is to notice however that some Italian speaking mathematics secondary teachers of Trieste based their teaching activity on a practical method founded on precise epistemological principles and cognitive theories arose in the Austrian cultural society.

3. The “Jacob Method”

In the second decade of the 20th century, when Trieste was still under the Habsburg Empire, Renato Marussig, a teacher which worked in the Italian speaking secondary schools of Trieste (first in the “old” *Gymnasium* and then in the “new” *Gymnasium-Realgymnasium* of Trieste since its foundation), translated into Italian two volumes of the textbook by Josef Jacob, written originally in German, for the teaching of arithmetic and algebra in the first classes of the *Gymnasium* (Jacob [7], [9]). This text was used in both Italian speaking *Gymnasien* of Trieste. Josef Jacob was an Austrian school director (in fact in 1912 he was the director of a *Gymnasium* in Vienna; see Wengraf [18]) who wrote, in particular, a book on a secondary school mathematics teaching method (Jacob [8]); a copy of this book is kept at the library of the “new” *Gymnasium-Realgymnasium* of Trieste, now called “*Ginnasio Liceo Francesco Petrarca*”.

This book contains a preface by Ernst Mach, the famous physicist and philosopher, who was a methodical scholar of the problems

of human perception. In his preface, Mach notices with pleasure, regarding his own *biological-economical theory of knowledge*, that “a practical and studious school man takes note of this theory and gets benefit from it for the school”.

Further, he stresses the importance of the psychological knowledge of the pupils and then examines briefly the subjects contained in the book; concerning differential and integral calculus, he expresses some doubts about the introduction of this subject in the secondary teaching, and says that he agrees with the book’s author in considering it “a further increase”. He concludes that “if we want to accept this subject, it may be indicated without timorous pedantry and without pomp of calculation”.

Jacob illustrates his “practical method for mathematics teaching” making precise references to Mach’s work. At the beginning of his own preface he observes that “the teaching plans for mathematics of 1908 and 1909 set several guiding principles whose correctness is recognized without exceptions, whereas the opinions about their execution are widely divergent”. He affirms that a practical teaching method needs a conscious scientific foundation and that his treatment is based almost exclusively on Mach’s point of view.

Then, he quotes some aspects of Mach’s knowledge theory which he intends to apply in mathematics teaching: the general principle of the economic function of every science (which substitutes the experience with representations of the facts), the idea about the origin and the aim of the science (which arises from real life necessities and comes back to these), and the idea that the kernels of the concepts are real situations.

Therefore he deduces the relevance of the following suggestions in mathematics teaching: to create representations, to give more value to mathematics through questions derived from real life, to care about by-hand activities. In the first section of the first chapter, regarding the aims of mathematics teaching, he details these considerations, with precise quotations to Mach’s works, first of all to *Erkenntnis und Irrtum* (Mach [11]).

4. The “Jacob Method” applied to differential and integral calculus

The section of Jacob’s book reserved to infinitesimal calculus begins with a polemic note about the new mathematics teaching programmes, which affirms that “they hide in a regrettable silence from the point of view of the goal, of the moment, and of the wideness [of the treatment of infinitesimal calculus in secondary schools]” .

Then the author answers to these open questions, with arguments based on the principles stated above. By considering that “the concepts are the instruments made by the human spirit for overcoming a field of knowledge” and affirming that the greatest applications of differential and integral calculus, in the secondary teaching, are in physics teaching, Jacob recognizes as a main goal for the teaching of infinitesimal calculus its application in the teaching of physics.

Nevertheless, the author says that it is not convenient to introduce these subjects during the introduction of the concept of velocity in physics, but before it. The reason is that “the instruments which are used for the construction of the knowledge field of other disciplines must be simple for the pupil in order to avoid complications”. Thus he proposes to introduce them in arithmetic by considering the question of increasing and decreasing of a function.

As far as the wideness of the treatment is concerned, he observes that the simplest applications of calculus to geometry and physics do not need particular theorems: this must be the range of the introduction of this subject, because “teaching is promoted only if it works as long as possible with concepts without theorems”. Finally, he observes that the application of calculus to physics [kinematics] needs both differential and integral calculus, whereas the new teaching plans prescribe only differential calculus in the *Gymnasium* and *Realgymnasium*.

After these general remarks, Jacob describes in detail a didactic way for introducing calculus at secondary school level, whose essential points are the following: he starts from a very intuitive introduction of the concept of limit of a function, illustrates the difference quotient and then the differential quotient (the derivative of the function) by considering the construction of the tangent to a curve (an algebraic curve), introduces the idea of integration as the “inverse

task” of differentiation, and finally explains the definite integral by calculating the area subtended by the graph of a function.

He proceeds step by step by means of examples and gives practical suggestions to the teacher, in particular about the symbols to use, i.e. he recommends to use the most familiar and simplest symbols for indicating the differences instead of others (such as Δx or Δy), and to avoid to indicate the indefinite integral of a function with the integral symbol, before introducing it for the definite integral.

5. The situation after the First World War

Not only Marussig, but also several secondary Italian speaking mathematics teachers of Trieste, who founded in 1919 (after the city of Trieste had been annexed to the Kingdom of Italy) the Trieste Section of the “Mathesis” Society,² esteemed the “Jacob Method” highly.

In fact, this method was the subject of an article by a member of this Section in *Periodico di matematiche* (Voghera [15]), entitled “On a method of teaching mathematics in use in the new provinces”. In this article (written in 1922) the author stresses the affinity of the “Jacob Method” with “the principles already long supported by Klein and also by all the other members of the International Commission on Mathematics Instruction, having as a goal the contemporary and harmonious development of the three faculty (i.e. intuitive, deductive, and creative) of the pupils, with regard to the capacities corresponding to their age”.

Further, in 1920 Marussig was invited by the colleagues to translate the third volume of Jacob’s textbook for secondary schools.³ Jacob-Marussig’s textbook was adopted in the “*Ginnasio Liceo Francesco Petrarca*” of Trieste until 1933,⁴ even if in 1923 a reform led by Minister Giovanni Gentile modified the school regulations and programmes in the whole Kingdom of Italy. This reform oriented the

²“Mathesis” — “Italian Society of Mathematics and Physics” — was founded in the Kingdom of Italy, in Turin, in 1895. Its aim was to improve and advance the teaching of mathematics and, more generally, of science teaching.

³Minutes of the General Congress of June 12, 1920, in *Fondo Mathesis*, 1.

⁴Archivio di Stato di Trieste, *Liceo ginnasio “Francesco Petrarca” di Trieste*, reg. no. 1 (minutes of teaching staff meetings), pp. 108, 150-151.

secondary instruction more towards classical studies to the detriment of scientific disciplines (see Giacardi [6]). In particular, it eliminated the *Liceo Moderno* and the *Physical-Mathematical Section* of the *Istituto Tecnico* (corresponding in Trieste to the former Austrian *Realschule*) and instituted the so-called *Liceo Scientifico*. In this type of school the teaching of differential calculus was provided, whereas in Italian *Liceo Classico* it continued not to be included.

Encouraged initially to propose their ideas for the gradual transition to the programmes of the Kingdom of Italy, while awaiting a reform that was supposed to take into consideration their experience (but did not really) the Trieste “Mathesis” teachers had presented their own recommendations for school programmes and for the direction mathematics instruction should take.

Mathematics programmes as prepared by the Trieste Section of “Mathesis” for the *Ginnasio-Liceo Classico* and *Moderno* (the Italian denominations of the *Gymnasium* and *Realgymnasium*) — the same programmes for both types of schools — were quite similar to those which had been in effect until then in Austria, though they included some elements from the Kingdom school programmes (see Zuccheri and Zudini [20]). In particular, as far as differential and integral calculus was concerned, the subject was maintained in both types of school.⁵

REFERENCES

- [1] M. ARTIGUE, *Learning and teaching analysis: What can we learn from the past in order to think about the future?*, in Coray et al. [3], 213–223.
- [2] E. BEKE, *Les résultats obtenus dans l'introduction du calcul différentiel et intégral dans les classes supérieures des établissements secondaires, Rapport Général présenté à la séance du 2 Avril 1914*, L'Enseignement Mathématique **16** (1914), 245–284.
- [3] D. CORAY, F. FURINGHETTI, H. GISPERT, B.R. HODGSON AND G. SCHUBRING eds., *One Hundred Years of L'Enseignement Mathématique. Moments of Mathematics Education in the Twentieth Century. Proceedings of the EM-ICMI Symposium*, Geneva, October 20–22 (2000), L'Enseignement Mathématique, Geneva (2003).

⁵*Fondo Mathesis*, 6.25.

- [4] F. FURINGHETTI, *Mathematical instruction in an international perspective: The contribution of the journal L'Enseignement Mathématique*, in Coray et al. [3], 19–46.
- [5] L. GIACARDI ed., *Da Casati a Gentile. Momenti di storia dell'insegnamento secondario della matematica in Italia*, Lumières Internationales, Lugano (2006).
- [6] L. GIACARDI, *L'insegnamento della matematica in Italia dall'Unità all'avvento del fascismo*, in Giacardi [5], 1–63.
- [7] J. JACOB, *Manuale d'Aritmetica per la prima classe delle scuole medie*, Quidde, Trieste (1912), Italian version by R. Marussig.
- [8] J. JACOB, *Praktische Methodik des mathematischen Unterrichts*, A. Pichlers Witwe & Sohn, Wien (1913), preface by E. Mach.
- [9] J. JACOB, *Manuale d'Aritmetica per la seconda classe delle scuole medie*, Quidde, Trieste (1914), Italian version by R. Marussig.
- [10] J.P. KAHANE, *L'enseignement du calcul différentiel et intégral au début du vingtième siècle*, in Coray et al. [3], 167–178.
- [11] E. MACH, *Erkenntnis und Irrtum: Skizzen zur Psychologie der Forschung*, Barth, Leipzig (1906), 2nd edition.
- [12] F. MOČNIK, *Trattato di Aritmetica e Algebra per le classi superiori delle scuole medie*, Trieste (1891), Italian version by E. Menegazzi.
- [13] F. MOČNIK AND K. ZAHRADNÍČEK, *Arithmetik und Algebra für die V. bis VIII. Klasse der Gymnasien und Realgymnasien*, F. Tempsky, Wien (1917).
- [14] G. SCHUBRING, *L'Enseignement Mathématique and the first International Commission (IMUK): The emergence of international communication and cooperation*, in Coray et al. [3], 47–65.
- [15] G. VOGHERA, *Intorno ad un metodo di insegnamento della matematica in uso nelle scuole delle terre redente*, Periodico di Matematiche: Storia - Didattica - Filosofia **II**, no. 5 (1922), 475–478.
- [16] F. WALLENTIN, *Manuale di Aritmetica per la terza e quarta classe delle scuole medie e degli altri istituti affini*, Stabilimento Tipogr. G.B. Monauni, Trento (1892), Italian version by F. Postet.
- [17] F. WALLENTIN, *Trattato di Aritmetica per le classi superiori dei ginnasi e delle scuole reali*, Stabilimento Tipogr. G.B. Monauni, Trento (1895), Italian version by F. Postet.
- [18] R. WENGRAF, *Jahrbuch des höheren Unterrichtswesens in Österreich*, F. Tempsky, Wien (1912).
- [19] L. ZUCCHERI AND V. ZUDINI, *Identity and culture in didactic choices made by mathematics teachers of the Trieste Section of "Mathesis" from 1918 to 1923*, International Journal for the History of Mathematics Education, to appear.
- [20] L. ZUCCHERI AND V. ZUDINI, *Animi divisi. Vicende dell'insegna-*

mento della matematica nella Venezia Giulia dal 1918 al 1923, EUT, Trieste (2007).

- [21] See also these school year-books of the following secondary schools of Trieste:
- *Civica Scuola Reale Superiore all'Acquedotto (Istituto Tecnico "G. Galilei")*, school years 1909–1910, 1911–1912 and from 1918 to 1923.
 - *Ginnasio Superiore Comunale (Ginnasio-Liceo) "Dante Alighieri"*, school years from 1905 to 1913 and from 1918 to 1922.
 - *Ginnasio Superiore Comunale (Ginnasio-Liceo) "Francesco Petrarca"*, school years from 1915 to 1922.
- [22] See also: *Fondo Mathesis*, kept at the Library of the Department of Mathematics and Computer Science, University of Trieste; Archivio di Stato di Trieste, "*Liceo ginnasio Francesco Petrarca*" di Trieste.

Received October 31, 2007.