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Exponential Stability of a Thermoelastic System Without Mechanical Dissipation

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Dedicated to Pierre Grisvard

Summary. - *We show herein the uniform stability of a thermoelastic plate model with no added dissipative mechanism on the boundary (uniform stability of a thermoelastic plate with added boundary dissipation was shown in [3], as was that of the analytic case–where rotational forces are neglected–in [8]); both the analytic and nonanalytic cases are treated here. The proof is constructive in the sense that we make use of a multiplier with respect to the coupled system involved so as to generate* a fortiori *the desired estimates; this multiplier is of an operator theoretic nature, as opposed to the more standard differential quantities used for such work. Moreover, the particular choice of multiplier becomes clear only after recasting the pde model into an associated abstract evolution equation. With this direct technique, we also obtain an exponential stability estimate pertaining to the limit case in which rotational inertia is neglected, and which leads to an associated analytic semigroup.*

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1. Introduction

1.1 Statement of the Problem

Let Ω be a bounded open subset of $\mathbb{R}^2$ with sufficiently smooth boundary Γ. We consider here the following thermoelastic system taken from J. Lagnese's monograph [3]:

$$\begin{cases} \begin{cases} \omega_{tt} - \gamma\Delta\omega_{tt} + \Delta^2\omega + \alpha\Delta\theta = 0 \\ \beta\theta_t - \eta\Delta\theta + \sigma\theta - \alpha\Delta\omega_t = 0 \end{cases} \quad \text{in } (0,\infty)\times\Omega; \\ \dfrac{\partial\theta}{\partial\nu} + \lambda\theta = 0 \qquad \text{on } (0,\infty)\times\Gamma,\ \lambda \geq 0; \\ \omega(t=0) = \omega^0,\ \omega_t(t=0) = \omega^1,\ \theta(t=0) = \theta^0 \text{ on } \Omega; \end{cases} \tag{1}$$

$$\begin{cases} \omega = (1-\kappa)\dfrac{\partial\omega}{\partial\nu} = 0 \\ \kappa\left(\Delta\omega + (1-\mu)B_1\omega + \alpha\theta\right) = 0 \end{cases} \quad \text{on } (0,\infty)\times\Gamma. \tag{2}$$

Here, the parameter κ is either 0 or 1; α, β and η are strictly positive constants; nonegative constant γ is proportional to the thickness of the plate and assumed to be small with $0 \leq \gamma \leq M$; the constant $\sigma \geq 0$ and the boundary operator B_1 is given by

$$B_1\omega \equiv 2\nu_1\nu_2\frac{\partial^2\omega}{\partial x\partial y} - \nu_1^2\frac{\partial^2\omega}{\partial y^2} - \nu_2^2\frac{\partial^2\omega}{\partial x^2}; \tag{3}$$

the constant μ is the familiar Poisson's ratio $\in (0, \frac{1}{2})$. The given model mathematically describes a Kirchoff plate – the displacement of which is represented by the function ω – subjected to a thermal damping as quantified by θ. We are concerned here with the asymptotic stability of solutions $[\omega, \theta]$ to (1)–(2).

1.2 Preliminaries and Abstract Formulation

As a departure point for obtaining the proofs of well-posedness and of exponential stability, we will consider the system (1)–(2) as an abstract evolution equation in a certain Hilbert space, for which we introduce the following definitions and notations.

- We define $\mathring{\mathbf{A}}_\kappa : L^2(\Omega) \supset D\left(\mathring{\mathbf{A}}_\kappa\right) \to L^2(\Omega)$ to be $\mathring{\mathbf{A}}_\kappa = \Delta^2$, with domain

$$D(\mathring{\mathbf{A}}_\kappa) = \left\{\omega \in H^4(\Omega) \cap H_0^1(\Omega) : \mathcal{B}\omega = 0 \text{ on } \Gamma\right\}, \qquad (4)$$

where

$$\mathcal{B}\omega = \begin{cases} \dfrac{\partial \omega}{\partial \nu}, \text{ if } \kappa = 0 \\ \\ \Delta\omega + (1-\mu)B_1\omega, \text{ if } \kappa = 1; \end{cases} \qquad (5)$$

- $\mathring{\mathbf{A}}_\kappa$ is then positive definite, self-adjoint, and consequently from [1] we have the characterizations

$$D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}) = \begin{cases} H_0^2(\Omega), \text{ if } \kappa = 0 \\ \\ H^2(\Omega) \cap H_0^1(\Omega), \text{ if } \kappa = 1; \end{cases}$$
$$D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{4}}) = H_0^1(\Omega). \qquad (6)$$

Moreover, using the Green's formula in [3], we have that for $\forall\, \omega, \widehat{\omega} \in D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})$,

$$\begin{aligned} \left\langle \mathring{\mathbf{A}}_\kappa \omega, \widehat{\omega} \right\rangle_{\left[D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})\right]' \times D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})} &= \left(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega, \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\widehat{\omega}\right)_{L^2(\Omega)} \\ &= a\left(\omega, \widehat{\omega}\right)_{L^2(\Omega)}, \end{aligned} \qquad (7)$$

where $a(\cdot,\cdot)$ is defined by

$$\begin{aligned} a\left(\omega, \widehat{\omega}\right) \equiv \int_\Omega [\omega_{xx}\widehat{\omega}_{xx} + \omega_{yy}\widehat{\omega}_{yy} + \mu\left(\omega_{xx}\widehat{\omega}_{yy} + \omega_{yy}\widehat{\omega}_{xx}\right) \\ + 2(1-\mu)\omega_{xy}\widehat{\omega}_{xy}]d\Omega, \end{aligned} \qquad (8)$$

and in addition

$$\|\omega\|^2_{D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})} = \left\|\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} = a\left(\omega, \omega\right). \qquad (9)$$

- We define $A_D : L^2(\Omega) \supset D(A_D) \to L^2(\Omega)$ to be $A_D = -\Delta$, with Dirichlet boundary conditions, viz.

$$D(A_D) = H^2(\Omega) \cap H_0^1(\Omega). \tag{10}$$

  A_D is also positive definite, self-adjoint, and

$$D(A_D^{\frac{1}{2}}) = H_0^1(\Omega). \tag{11}$$

- $A_R : L^2(\Omega) \supset D(A_R) \to L^2(\Omega)$ will designate the following second order elliptic operator:

$$\begin{aligned} A_R &= -\Delta + \frac{\sigma}{\eta}\mathbf{I}, \\ D(A_R) &= \left\{ \theta \in H^2(\Omega) : \frac{\partial \theta}{\partial \nu} + \lambda\theta = 0 \right\}; \end{aligned} \tag{12}$$

  A_R is positive definite, self-adjoint, and once more by the characterization of the fractional powers in [1], and we have

$$\left(\nabla\theta, \nabla\widetilde{\theta}\right)_{L^2(\Omega)} + \lambda\left(\theta, \widetilde{\theta}\right)_{L^2(\Gamma)} + \frac{\sigma}{\eta}\left(\theta, \widetilde{\theta}\right)_{L^2(\Omega)} = \left(= A_R^{\frac{1}{2}}\theta, A_R^{\frac{1}{2}}\widetilde{\theta}\right)_{L^2(\Omega)}. \tag{13}$$

  Thus, if $\sigma + \lambda > 0$, we have the topological equivalence

$$\left(\theta, \widetilde{\theta}\right)_{H^1(\Omega)} \cong \left(A_R^{\frac{1}{2}}\theta, A_R^{\frac{1}{2}}\widetilde{\theta}\right)_{L^2(\Omega)}. \tag{14}$$

- We will designate by γ_0 the Sobolev trace map, whichs yields for $f \in C^\infty(\overline{\Omega})$

$$\gamma_0 f = f|_\Gamma . \tag{15}$$

- We define the elliptic operators G and D as thus:

$$Gh = v \Longleftrightarrow \begin{cases} \Delta^2 v = 0 \text{ in } (0,\infty)\times\Omega \\ \begin{cases} v|_\Gamma = 0 \\ \Delta v + (1-\mu)B_1 v = h \end{cases} \text{ on } (0,\infty)\times\Gamma; \end{cases} \tag{16}$$

$$Dh = v \Longleftrightarrow \begin{cases} \Delta v = 0 \ \text{ on } (0,\infty)\times\Omega \\ \\ v|_{\Gamma} = h \ \text{ on } (0,\infty)\times\Gamma. \end{cases} \tag{17}$$

The classic regularity results of [4] then provide that for $s \in \mathbb{R}$,

$$\begin{cases} D \in \mathcal{L}\left(H^s(\Gamma), H^{s+\frac{1}{2}}(\Omega)\right) \\ G \in \mathcal{L}\left(H^s(\Gamma), H^{s+\frac{5}{2}}(\Omega)\right). \end{cases} \tag{18}$$

With the operators $\mathring{\mathbf{A}}_1$ and G as defined above, one can readily show with the use of Green's formula that $\forall \omega \in D(\mathring{\mathbf{A}}_1^{\frac{1}{2}})$ the adjoint $G^*\mathring{\mathbf{A}}_1 \ \in \mathcal{L}\Big(D(\mathring{\mathbf{A}}_1^{\frac{1}{2}}), L^2(\Gamma)\Big)$ satisfies

$$G^*\mathring{\mathbf{A}}_1\omega = \left.\frac{\partial\omega}{\partial\nu}\right|_{\Gamma}; \tag{19}$$

- We define the operator P_γ by

$$P_\gamma \equiv \mathbf{I} + \gamma A_{D,} \tag{20}$$

and:

(i) *In the case* $\gamma > 0$, we define a space $H^1_{0,\gamma}(\Omega)$ equivalent to $H^1_0(\Omega)$ with its inner product as

$$(\omega_1,\omega_2)_{H^1_{0,\gamma}(\Omega)} \equiv (\omega_1,\omega_2)_{L^2(\Omega)} + \gamma\,(\nabla\omega_1,\nabla\omega_2)_{L^2(\Omega)} \qquad \forall\,\omega_1,\omega_2 \in H^1_0(\Omega), \tag{21}$$

and with its dual denoted as $H^{-1}_\gamma(\Omega)$. (11) then yields that

$$P_\gamma \in \mathcal{L}\left(H^1_{0,\gamma}(\Omega), H^{-1}_\gamma(\Omega)\right), \text{ with} \tag{22}$$

$$\langle P_\gamma\omega_1,\omega_2\rangle_{H^{-1}_\gamma(\Omega)\times H^1_{0,\gamma}(\Omega)} = (\omega_1,\omega_2)_{H^1_{0,\gamma}(\Omega)}. \tag{23}$$

Furthermore, the $H^1_{0,\gamma}(\Omega)$-ellipticity of P_γ and Lax-Milgram gives us that P_γ is boundedly invertible, i.e.

$$P_\gamma^{-1} \in \mathcal{L}\left(H^{-1}_\gamma(\Omega), H^1_{0,\gamma}(\Omega)\right). \tag{24}$$

Finally, P_γ being positive definite, self-adjoint as an operator $P_\gamma : L^2(\Omega) \supset D(P_\gamma) \to L^2(\Omega)$ (as A_D is), the square root $P_\gamma^{\frac{1}{2}}$ is well-defined with $D(P_\gamma^{\frac{1}{2}}) = H^1_{0,\gamma}(\Omega)$ (using the interpolation theorem in [4], p. 10); it then follows from (23) that for ω and $\widehat{\omega} \in H^1_{0,\gamma}(\Omega)$,

$$\left\| P_\gamma^{\frac{1}{2}} \omega \right\|^2_{L^2(\Omega)} = \|\omega\|^2_{L^2(\Omega)} + \gamma \|\nabla \omega\|^2_{L^2(\Omega)} = \|\omega\|^2_{H^1_{0,\gamma}(\Omega)} \tag{25}$$

$$\left(P_\gamma^{\frac{1}{2}} \omega, P_\gamma^{\frac{1}{2}} \widehat{\omega} \right)_{L^2(\Omega)} = (\omega, \widehat{\omega})_{H^1_{0,\gamma}(\Omega)} . \tag{26}$$

(ii) *In the case* $\gamma = 0$, (20) gives that $P_0 = \mathbf{I}$, and we simply set the spaces

$$H^1_{0,0}(\Omega) = H_0^{-1}(\Omega) \equiv L^2(\Omega). \tag{27}$$

- With $L^2_{\sigma+\lambda}(\Omega)$ defined by

$$L^2_{\sigma+\lambda}(\Omega) \equiv \begin{cases} L^2(\Omega), & \text{if } \sigma + \lambda > 0 \\ L^2_0(\Omega), & \text{if } \sigma + \lambda = 0 \end{cases} \tag{28}$$

(where $L^2_0(\Omega) = \left\{ \theta \in L^2(\Omega) \ni \int_\Omega \theta = 0 \right\}$), we denote the Hilbert space $\mathbf{H}_{\kappa,\gamma}$ to be

$$\mathbf{H}_{\kappa,\gamma} \equiv D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}) \times H^1_{0,\gamma}(\Omega) \times L^2_{\sigma+\lambda}(\Omega), \tag{29}$$

with the inner product

$$\left(\begin{bmatrix} \omega_1 \\ \omega_2 \\ \theta \end{bmatrix}, \begin{bmatrix} \widehat{\omega}_1 \\ \widehat{\omega}_2 \\ \widehat{\theta} \end{bmatrix} \right)_{\mathbf{H}_{\kappa,\gamma}} = \left(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}} \omega_1, \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}} \widehat{\omega}_1 \right)_{L^2(\Omega)} + \left(P_\gamma^{\frac{1}{2}} \omega_2, P_\gamma^{\frac{1}{2}} \widehat{\omega}_2 \right)_{L^2(\Omega)} + \beta \left(\theta, \widehat{\theta} \right)_{L^2(\Omega)} . \tag{30}$$

With the above definitions, we then set $\mathcal{A}_{\kappa,\gamma} : \mathbf{H}_{\kappa,\gamma} \supset D(\mathcal{A}_{\kappa,\gamma}) \to \mathbf{H}_{\kappa,\gamma}$ to be

$$\mathcal{A}_{\kappa,\gamma} \equiv \begin{pmatrix} 0 & \mathbf{I} & 0 \\ -P_\gamma^{-1}\mathring{\mathbf{A}}_\kappa & 0 & \begin{array}{r}\alpha P_\gamma^{-1}(A_D(\mathbf{I}-D\gamma_0)- \\ -\,\kappa\mathring{\mathbf{A}}_1 G\gamma_0)\end{array} \\ 0 & -\dfrac{\alpha}{\beta}A_D & -\dfrac{\eta}{\beta}A_D(\mathbf{I}-D\gamma_0)-\dfrac{\sigma}{\beta}\mathbf{I} \end{pmatrix}$$

$$D(\mathcal{A}_{\kappa,\gamma}) = \left\{ \begin{array}{l} [\omega_1,\omega_2,\theta] \in D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}) \times D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}) \times D(A_R) \cap L^2_{\sigma+\lambda}(\Omega) \\ \text{such that } \mathring{\mathbf{A}}_\kappa\omega_1 + \alpha\kappa\mathring{\mathbf{A}}_1 G\gamma_0\theta \in H_\gamma^{-1}(\Omega) \\ \text{and } \dfrac{\alpha}{\beta}\Delta\omega_2 + \dfrac{\eta}{\beta}\Delta\theta \in L^2_{\sigma+\lambda}(\Omega) \end{array} \right\}. \tag{31}$$

We note that $\left[\frac{\eta}{\beta}A_D(\mathbf{I}-D\gamma_0)+\frac{\sigma}{\beta}\right]\theta = -\frac{\eta}{\beta}\Delta\theta + \frac{\sigma}{\beta}\theta = A_R\theta$ for $\theta \in D(A_R)$.

If we take the initial data $\left[\omega^0,\omega^1,\theta^0\right]$ to be in $\mathbf{H}_{\kappa,\gamma}$, then the coupled system (1)–(2) becomes the operator theoretic model

$$\begin{gathered} \frac{d}{dt}\begin{bmatrix} \omega \\ \omega_t \\ \theta \end{bmatrix} = \mathcal{A}_{\kappa,\gamma}\begin{bmatrix} \omega \\ \omega_t \\ \theta \end{bmatrix} \\ \begin{bmatrix} \omega(0) \\ \omega_t(0) \\ \theta(0) \end{bmatrix} = \begin{bmatrix} \omega^0 \\ \omega^1 \\ \theta^0 \end{bmatrix}. \end{gathered} \tag{32}$$

REMARK 1. For initial data $\left[\omega^0,\omega^1,\theta^0\right]$ in $D(\mathcal{A}_{\kappa,\gamma})$, the two equations of (1) may be written pointwise as

$$P_\gamma\omega_{tt} = -\mathring{\mathbf{A}}_\kappa\omega - \alpha\kappa\mathring{\mathbf{A}}_1 G\gamma_0\theta + \alpha A_D(I-D\gamma_0)\theta \text{ in } H_\gamma^{-1}(\Omega); \tag{33}$$

$$\beta\theta_t = -\eta A_D(\mathbf{I}-D\gamma_0)\theta - \sigma\theta - \alpha A_D\omega_t \text{ in } L^2_{\sigma+\lambda}(\Omega). \tag{34}$$

1.3 Previous Literature

In [3], J. Lagnese established the well-posedness and exponential stability of (1) with γ strictly positive, and with the following B.C.'s replacing those of (2):

$$\begin{cases} \omega = \dfrac{\partial \omega}{\partial \nu} = 0 \text{ on } (0,\infty) \times \Gamma_0 \\ \Delta\omega + (1-\mu)B_1\omega + \alpha\theta = \mathcal{F}_1(\omega_t) \text{ on } (0,\infty)\times\Gamma_1 \\ \dfrac{\partial \Delta\omega}{\partial \nu} + (1-\mu)\dfrac{\partial B_2\omega}{\partial \tau} - \gamma\dfrac{\partial \omega_{tt}}{\partial \nu} + \alpha\dfrac{\partial \theta}{\partial \nu} = \mathcal{F}_2(\omega_t) \text{ on } (0,\infty)\times\Gamma_1, \end{cases} \tag{35}$$

where $\Gamma = \Gamma_0 \cup \Gamma_1$, with $\overline{\Gamma}_0 \cap \overline{\Gamma}_1 = \emptyset$, $\Gamma_0 \neq \emptyset$, and $\mathcal{F}_1(\omega_t)$, $\mathcal{F}_2(\omega_t)$ are appropriately chosen dissipative feedbacks; the proof of Lagnese is based on the use of differential multipliers, and it exploits the fact that $\gamma > 0$. Since, from a physical point of view, the thermal effects present should induce some measure of energy dissipation (in fact, one can show the system's strong stability by routine methods), a natural question arising in this context is whether the system is actually (uniformly) stable without the boundary feedbacks $\mathcal{F}_1(\omega_t)$, $\mathcal{F}_2(\omega_t)$ in place, i.e. when there are no added mechanical forces. Indeed, in the case $\gamma = 0$, the answer to the question is in the affirmative and has been provided by several authors. With $\gamma = 0$, Kim [2] showed the uniform stability of (1) with the boundary conditions $\omega = \dfrac{\partial\omega}{\partial\nu} = \theta = 0$ on Γ, as did Rivera and Racke in [9] with the boundary conditions $\omega = \Delta\omega = \theta = 0$. Also with $\gamma = 0$, Liu and Zheng in [8] proved the exponential stability of (1) with the boundary conditions

$$\begin{cases} \omega = \dfrac{\partial \omega}{\partial \nu} = 0 \text{ on } (0,\infty)\times\Gamma_0 \\ \omega = \Delta\omega + (1-\mu)B_1\omega + \alpha\theta = 0 \text{ on } (0,\infty)\times\Gamma_1, \end{cases} \tag{36}$$

where Γ_0 and Γ_1 are as in (35). The proof of Liu and Zheng is indirect in the sense that it is based on a contradiction argument applied to the exponential decay stability criterion (due to L.A. Monauni, R.

Nagel and F.L. Huang), a criterion essentially dictating the uniform estimate for that part of the resolvent which lies on the imaginary axis. On the other hand, it is now known that the case $\gamma = 0$ is rather special as the corresponding system generates an *analytic* semigroup (see [7], [12]), a consequence of which will be the exponential stability of the system (recall that the system is strongly stable). Given these results, the question of interest now is whether the given thermoelastic system (without any additional boundary dissipation) is *uniformly* stable in the *nonanalytic* case, viz. $\gamma > 0$, with consequently the elastic part of the system being of hyperbolic character. (It is known [11] that the case $\gamma > 0$ is not analytic.)

The main goal of this paper is to provide an affirmative answer to the question posed above, pertaining to the case $\gamma > 0$. In fact, we shall show that the energy of (1) decays exponentially to zero with accompanying rates which are *uniform* with respect to the parameter $\gamma \geq 0$. (The case of free boundary condition is studied in [10].)

In this way, the stability result for $\gamma = 0$ is reconstructed, although the cases $\gamma > 0$ and $\gamma = 0$ correspond respectively to very different dynamics (hyperbolic versus parabolic). Our proof is "direct", based on pseudodifferential (or operator theoretic) multipliers, in contrast to the contradiction argument supplied in [8], valid for $\gamma = 0$. Another advantage of the "direct" proof provided herein is that it leads to explicit estimates of the decay rates. A preliminary version of the present paper is given in [13].

1.4 Statement of the Results

We shall begin by giving preliminary results regarding the well-posedness of the system (1)–(2) and the regularity of its solutions.

THEOREM 1. *(well-posedness) Again with the parameters κ either 0 or 1 and $\gamma \geq 0$, $\mathcal{A}_{\kappa,\gamma}$, given by (31), generates a C_0-semigroup of contractions $\left\{e^{\mathcal{A}_{\kappa,\gamma} t}\right\}_{t\geq 0}$ on the energy space $\boldsymbol{H}_{\kappa,\gamma}$; therefore for initial data $\left[\omega^0, \omega^1, \theta^0\right] \in \boldsymbol{H}_{\kappa,\gamma}$, the solution $[\omega, \omega_t, \theta]$ to (32), and consequently to (1)–(2) is given by*

$$\begin{bmatrix} \omega \\ \omega_t \\ \theta \end{bmatrix} = e^{\mathcal{A}_{\kappa,\gamma} t} \begin{bmatrix} \omega^0 \\ \omega^1 \\ \theta^0 \end{bmatrix}. \tag{37}$$

The following regularity result is needed to justify the computations performed below.

THEOREM 2. ***(i)*** *For initial data* $\left[\omega^0,\omega^1,\theta^0\right] \in D\left(\mathcal{A}^2_{\kappa,\gamma}\right)$, *we have that the solution* $[\omega,\omega_t,\theta]$ *to (1)–(2) satisfies* $\omega \in C([0,T];\, H^4(\Omega))$, $\omega_t \in C([0,T];\, H^3(\Omega))$ *and* $\theta \in C([0,T];\, H^3(\Omega))$.
(ii) $\omega + \alpha\kappa G\gamma_0\theta \in C([0,T];\, D(\mathring{\boldsymbol{A}}_\kappa))$.

Our main result is:

THEOREM 3. *(uniform stability) With* $\kappa = 0$ *or* 1 *and* $\gamma \geq 0$, *the solution* $[\omega,\omega_t,\theta]$ *of (1)–(2) decays exponentially; that is, there exist constants* $\delta > 0$ *and* $M_\delta \geq 1$ *(independent of* κ *and* γ*) such that for all* $t > 0$

$$\left\| \begin{bmatrix} \omega(t) \\ \omega_t(t) \\ \theta(t) \end{bmatrix} \right\|_{\boldsymbol{H}_{\kappa,\gamma}} \leq M_\delta e^{-\delta t} \left\| \begin{bmatrix} \omega^0 \\ \omega^1 \\ \theta^1 \end{bmatrix} \right\|_{\boldsymbol{H}_{\kappa,\gamma}} . \tag{38}$$

REMARK 2. The same result (and the same proof) holds when partially clamped boundary conditions replace those of (2).

As mentioned above, we will prove **Theorem 3** by explicitly applying a suitable operator theoretic multiplier.

2. Proofs

The proofs of well-posedness and of regularity (Theorems 1-2) are by now fairly routine (see Chap. 7 in [3] for related well-posedness / regularity results). However, since these preliminaries are critical for our ultimate end of uniform stability, we provide their concise proofs for the sake of completeness.

2.1 Proof of Theorem 1

In establishing the semigroup generation of $\mathcal{A}_{\kappa,\gamma}$, we will show that the conditions of the Lumer-Phillips Theorem are satisfied; namely, we demonstrate here that $\mathcal{A}_{\kappa,\gamma}$ is maximal dissipative.

To show the dissipativity of $\mathcal{A}_{\kappa,\gamma}$: For $[\omega_1,\omega_2,\theta] \in D(\mathcal{A}_{\kappa,\gamma})$ we have

$$\left(\mathcal{A}_{\kappa,\gamma}\begin{bmatrix}\omega_1\\ \omega_2\\ \theta\end{bmatrix},\begin{bmatrix}\omega_1\\ \omega_2\\ \theta\end{bmatrix}\right)_{\mathbf{H}_{\kappa,\gamma}} = \left(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_2, \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_1\right)_{L^2(\Omega)} +$$
$$+\left(P_\gamma^{\frac{1}{2}}P_\gamma^{-1}(-\mathring{\mathbf{A}}_\kappa\omega_1 + \alpha A_D(\mathbf{I}-D\gamma_0)\theta\right.$$
$$\left. -\alpha\kappa\mathring{\mathbf{A}}_1 G\gamma_0\theta), P_\gamma^{\frac{1}{2}}\omega_2\right)_{L^2(\Omega)}$$
$$-\alpha\left(A_D\omega_2,\theta\right)_{L^2(\Omega)} - \left(\eta A_D(\mathbf{I}-D\gamma_0)\theta + \sigma\theta,\theta\right)_{L^2(\Omega)}; \quad (39)$$

Using the standard result that

$$\langle\omega^*,\omega\rangle_{H_\gamma^{-1}(\Omega)\times H_\gamma^1(\Omega)} = \langle\omega^*,\omega\rangle_{\left[D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})\right]'\times D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})} \quad (40)$$

for every $\omega^* \in H_\gamma^{-1}(\Omega)$ and $\omega \in D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})$, we have upon taking adjoints and using the characterization (19) in the second term on the RHS of (39),

$$\begin{aligned}(39) \quad = \quad & \left(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_2, \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_1\right)_{L^2(\Omega)} - \left\langle\mathring{\mathbf{A}}_\kappa\omega_1,\omega_2\right\rangle_{\left[D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})\right]'\times D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})}\\
& +\alpha\left(A_D(\mathbf{I}-D\gamma_0)\theta,\omega_2\right)_{L^2(\Omega)} - \alpha\kappa\left(\theta,\frac{\partial\omega_2}{\partial\nu}\right)_{L^2(\Gamma)}\\
& -\alpha\left(A_D\omega_2,\theta\right)_{L^2(\Omega)} - \left(\eta A_D(\mathbf{I}-D\gamma_0)\theta+\sigma\theta,\theta\right)_{L^2(\Omega)}\\
= \quad & \left(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_2, \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_1\right)_{L^2(\Omega)} - \left(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_1, \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_2\right)_{L^2(\Omega)}\\
& -\alpha\left(\Delta\theta,\omega_2\right)_{L^2(\Omega)} - \alpha\kappa\left(\theta,\frac{\partial\omega_2}{\partial\nu}\right)_{L^2(\Gamma)}\\
& +\alpha\left(\Delta\omega_2,\theta\right)_{L^2(\Omega)} + \left(\eta\Delta\theta-\sigma\theta,\theta\right)_{L^2(\Omega)}\end{aligned}$$

$$
\begin{aligned}
&= \left(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_2, \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_1\right)_{L^2(\Omega)} - \left(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_1, \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega_2\right)_{L^2(\Omega)} \\
&\quad +\alpha\left(\nabla\theta, \nabla\omega_2\right)_{L^2(\Omega)} - \alpha\left(\nabla\omega_2, \nabla\theta\right)_{L^2(\Omega)} \\
&\quad -\eta\left\|\nabla\theta\right\|^2_{L^2(\Omega)} - \lambda\eta\left\|\theta\right\|^2_{L^2(\Gamma)} - \sigma\left\|\theta\right\|^2_{L^2(\Omega)} \\
&\leq 0; \qquad (41)
\end{aligned}
$$

i.e. $\mathcal{A}_{\kappa,\gamma}$ is dissipative.

To show the maximality of $\mathcal{A}_{\kappa,\gamma}$: if for some $\xi > 0$ and arbitrary $[f_1, f_2, f_3] \in \mathbf{H}_{\kappa,\gamma}$, $[\omega_1, \omega_2, \theta] \in D(\mathcal{A}_{\kappa,\gamma})$ solves the equation

$$
(\xi\mathbf{I} - \mathcal{A}_{\kappa,\gamma}) \begin{bmatrix} \omega_1 \\ \omega_2 \\ \theta \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}, \qquad (42)
$$

then this relation holds if and only if

$$
\begin{cases}
\xi\omega_1 - \omega_2 = f_1 \quad \text{in } D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}), \\
\xi\omega_2 + P_\gamma^{-1}\left(\mathring{\mathbf{A}}_\kappa\omega_1 + \alpha\kappa\mathring{\mathbf{A}}_1 G\gamma_0\theta - \alpha A_D(\mathbf{I} - D\gamma_0)\theta\right) = f_2 \\
\qquad\qquad\qquad \text{in } H^1_{0,\gamma}(\Omega), \\
\xi\theta + \dfrac{\alpha}{\beta}A_D\omega_2 + \dfrac{\eta}{\beta}A_D(\mathbf{I} - D\gamma_0)\theta + \dfrac{\sigma}{\beta}\theta = f_3 \quad \text{in } L^2_{\sigma+\lambda}(\Omega)
\end{cases} \qquad (43)
$$

$$
\Longleftrightarrow
$$

$$
\begin{cases}
\xi^3 P_\gamma\omega_1 + \xi\mathring{\mathbf{A}}_\kappa\omega_1 + \alpha\kappa\xi\mathring{\mathbf{A}}_1 G\gamma_0\theta - \alpha\xi A_R\theta + \dfrac{\alpha\xi\sigma}{\eta}\theta = \\
\qquad = \xi P_\gamma f_2 + \xi^2 P_\gamma f_1 \quad \text{in } H^{-1}_\gamma(\Omega), \\
\beta\xi\theta + \alpha\xi A_D\omega_1 + \eta A_R\theta = \beta f_3 + \alpha A_D f_1 \quad \text{in } L^2(\Omega)
\end{cases} \qquad (44)
$$

(given that $\theta \in D(A_R)$ as defined in (12)). At this point we bring forth the following:

Proposition 1. *The operator* $\boldsymbol{F}$ *defined by*

$$\boldsymbol{F} \equiv \begin{bmatrix} \xi^3 P_\gamma + \xi \mathring{\boldsymbol{A}}_\kappa & \alpha\kappa\xi \mathring{\boldsymbol{A}}_1 G\gamma_0 - \alpha\xi A_R + \dfrac{\alpha\xi\sigma}{\eta}\boldsymbol{I} \\ \alpha\xi A_D & \beta\xi \boldsymbol{I} + \eta A_R \end{bmatrix}, \qquad (45)$$

is an element of

$$\mathcal{L}\Big(D(\mathring{\boldsymbol{A}}_\kappa^{\frac{1}{2}}) \times H^1(\Omega) \cap L^2_\sigma(\Omega), \left[D(\mathring{\boldsymbol{A}}_\kappa^{\frac{1}{2}})\right]^{/} \times \left[H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)\right]^{/}\Big)$$

and is boundedly invertible.

Proof of Proposition 1. We first note (by Green's Theorem) that for arbitrary $\theta \in D(A_R)$ and $\omega \in D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})$,

$$\langle A_R\theta, \omega \rangle_{\left[D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})\right]^{/} \times D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})} = (\nabla\theta, \nabla\omega)_{L^2(\Omega)} + \frac{\sigma}{\eta}(\theta, \omega)_{L^2(\Omega)}; \qquad (46)$$

the characterization (14) and an extension by continuity will then have that (46) holds $\forall\, \theta \in H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)$. (46) in turn, when coupled with (13), (23) and (19), will yield the asserted boundedness of $\mathbf{F}$, and moreover (40), (23), (13), (19), (46) and Green's formula will provide the following coercivity inequality for all $[\omega, \theta] \in D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}) \times H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)$:

$$\begin{aligned}
&\left\langle \mathbf{F}\begin{bmatrix} \omega \\ \theta \end{bmatrix}, \begin{bmatrix} \omega \\ \theta \end{bmatrix} \right\rangle = \\
&= \xi^3 \|\omega\|^2_{L^2(\Omega)} + \xi^3\gamma \|\nabla\omega\|^2_{L^2(\Omega)} + \xi \left\| \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega \right\|^2_{L^2(\Omega)} \\
&\quad -\alpha\xi(\nabla\theta, \nabla\omega)_{L^2(\Omega)} + \alpha\xi(\nabla\theta, \nabla\omega)_{L^2(\Omega)} \\
&\quad -\frac{\sigma}{\eta}(\theta, \omega)_{L^2(\Omega)} + \frac{\sigma}{\eta}(\theta, \omega)_{L^2(\Omega)} + \\
&\quad +\eta\|\nabla\theta\|^2_{L^2(\Omega)} + \lambda\eta\|\theta\|^2_{L^2(\Gamma)} + \beta\xi\|\theta\|^2_{L^2(\Omega)} \geq \\
&\quad \text{(note the cancellation of boundary terms)} \\
&\geq C(\eta, \lambda, \xi)\left[\left\| \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega \right\|^2_{L^2(\Omega)} + \|\theta\|^2_{H^1(\Omega)\cap L^2_{\sigma+\lambda}(\Omega)} \right] \qquad (47)
\end{aligned}$$

(where $\langle \cdot, \cdot \rangle$ in (47) denotes the pairing between $D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}) \times H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)$ and its dual, the constant $C(\eta, \lambda, \xi) > 0$). Thus, by Lax-Milgram, $\mathbf{F}^{-1}$ exists as an element of

$$\mathcal{L}\Big(\Big[D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})\Big]' \times \Big[H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)\Big]', D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}) \times H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)\Big)$$

and the Proposition is proved. □

To complete the proof of the maximality of $\mathcal{A}_{\kappa,\gamma}$, we apply the inverse assured by **Proposition 1** to both sides of (44) to obtain

$$\begin{cases} \begin{bmatrix} \omega_1 \\ \theta \end{bmatrix} \equiv \mathbf{F}^{-1} \begin{bmatrix} \xi P_\gamma f_2 + \xi^2 P_\gamma f_1 \\ \beta f_3 + \alpha A_D f_1 \end{bmatrix} \\ \\ \omega_2 \equiv \xi \omega_1 - f_1, \end{cases} \tag{48}$$

and *a fortiori*, one has, by using the second equation in (44), that

$$A_R \theta = -\frac{\beta \xi}{\eta} \theta - \frac{\alpha \xi}{\eta} A_D \omega_1 + \frac{\beta}{\eta} f_3 + \frac{\alpha}{\eta} A_D f_1 \ \in L^2(\Omega),$$

viz. $\theta \in D(A_R) \cap L^2_{\sigma+\lambda}(\Omega)$. This additional regularity of θ, in conjunction with that implied in the first equation of (44), and along with the inclusion given in the third equation of (43), gives that our constructively acquired solution $[\omega_1, \omega_2, \theta]$ to (42) is in $D(\mathcal{A}_{\kappa,\gamma})$ as defined in (31). Hence, $\mathcal{A}_{\kappa,\gamma}$ is maximal dissipative and the proof of **Theorem 1** is complete.

2.2 Proof of Theorem 2

By definition, if $\left[\omega^0, \omega^1, \theta^0\right] \in D\left(\mathcal{A}_{\kappa,\gamma}\right)$, then $\omega^1 \in D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}})$, $\theta^0 \in D(A_R)$, and it is straightforward to show that

$$\mathring{\mathbf{A}}_\kappa \omega^0 + \alpha\kappa \mathring{\mathbf{A}}_1 G \gamma_0 \theta^0 = g \in H_\gamma^{-1}(\Omega) \subseteq \Big[D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{4}})\Big]' \tag{49}$$

(where if γ is strictly positive, the above containment into $\Big[D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{4}})\Big]'$ is actually equality); as $\mathring{\mathbf{A}}_\kappa^{-1} : \Big[D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{4}})\Big]' \to D(\mathring{\mathbf{A}}_\kappa^{\frac{3}{4}}) \subset H^3(\Omega)$ (this last containment deduced by the elliptic characterizations in [1]), we have after applying $\mathring{\mathbf{A}}_\kappa^{-1}$ to (49), the use of trace theory and the regularity posted in (18) that

$$\omega^0 = \mathring{\mathbf{A}}_\kappa^{-1} g - \alpha\kappa G\gamma_0\theta^0 \in H^3(\Omega). \tag{50}$$

Thus, for $\left[\omega^0, \omega^1, \theta^0\right] \in D\left(\mathcal{A}_{\kappa,\gamma}^2\right)$,

$$\mathcal{A}_{\kappa,\gamma}\begin{bmatrix}\omega^0\\ \omega^1\\ \theta^0\end{bmatrix} = \begin{bmatrix}\omega^1\\ \\ -P_\gamma^{-1}\mathring{\mathbf{A}}_\kappa\omega^0 - \alpha\kappa P_\gamma^{-1}\mathring{\mathbf{A}}_1 G\gamma_0\theta^0 + \\ + \alpha P_\gamma^{-1}A_D(\mathbf{I} - D\gamma_0)\theta^0 \\ \\ -\frac{\eta}{\beta}A_D(\mathbf{I} - D\gamma_0)\theta^0 - \\ -\frac{\sigma}{\beta}\theta^0 - \frac{\alpha}{\beta}A_D\omega^1\end{bmatrix} \in D\left(\mathcal{A}_{\kappa,\gamma}\right), \tag{51}$$

and (51) coupled with (50) implies that

$$\omega^1 \in H^3(\Omega). \tag{52}$$

In addition, the last component on the RHS of (51), (12), and (52) give that

$$A_R\theta^0 = h + \alpha\Delta\omega^1 \in H^1(\Omega), \tag{53}$$

where $h \in H^2(\Omega)$; applying ${A_R}^{-1}$ to both sides of (53) thus yields

$$\theta^0 \in H^3(\Omega). \tag{54}$$

Moreover, (51) also has

$$P_\gamma^{-1}\mathring{\mathbf{A}}_\kappa\omega^0 + \alpha\kappa P_\gamma^{-1}\mathring{\mathbf{A}}_1 G\gamma_0\theta^0 = g + \alpha P_\gamma^{-1}A_D(\mathbf{I} - D\gamma_0)\theta^0 \tag{55}$$

where $g \in D(\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}) \subset D(A_D)$, or equivalently

$$\mathring{\mathbf{A}}_\kappa\omega^0 + \alpha\kappa\mathring{\mathbf{A}}_1 G\gamma_0\theta^0 = g + \gamma\Delta g - \alpha\Delta\theta^0 \in L^2(\Omega). \tag{56}$$

A fortiori then, $\omega^0 + \alpha\kappa G\gamma_0\theta^0 \in D(\mathring{\mathbf{A}}_\kappa) \subset H^4(\Omega)$. But trace theory and the smoothing specified in (18) give that $G\gamma_0\theta^0 \in H^5(\Omega)$, and thus $D(\mathcal{A}_{\kappa,\gamma}^2) \subset H^4(\Omega) \times H^3(\Omega) \times H^3(\Omega)$ with continuous inclusion.

The solution $[\omega, \omega_t, \theta]$ will consequently have the asserted regularity upon consideration of the fundamental property that for $\xi \geq 0$, $[\omega^0, \omega^1, \theta^0] \in D\left(\mathcal{A}^{\xi}_{\kappa,\gamma}\right) \Rightarrow$

$$\begin{bmatrix} \omega \\ \omega_t \\ \theta \end{bmatrix} = e^{\mathcal{A}_{\kappa,\gamma}(\cdot)} \begin{bmatrix} \omega^0 \\ \omega^1 \\ \theta^0 \end{bmatrix} \in C\left([0,T]; D\left(\mathcal{A}^{\xi}_{\kappa,\gamma}\right)\right) . \tag{57}$$

To prove **(ii)**, we note that with $[\omega^0, \omega^1, \theta^0] \in D\left(\mathcal{A}^{2}_{\kappa,\gamma}\right)$, $\omega_{tt} \in C\left([0,T]; D(\mathring{\mathbf{A}}^{\frac{1}{2}}_{\kappa})\right)$, so the solution $[\omega, \omega_t, \theta]$ to (1) satisfies

$$-\mathring{\mathbf{A}}_{\kappa}\omega - \alpha\kappa\mathring{\mathbf{A}}_1 G\gamma_0\theta = \omega_{tt} - \gamma\Delta\omega_{tt} - \alpha\Delta\theta \tag{58}$$

in $C([0,T];\, L^2(\Omega))$, which establishes the result. □

REMARK 3. Because of the regularity result posted in Theorem 2 (ii), we have for sufficiently smooth initial data the valid representation

$$\mathring{\mathbf{A}}_{\kappa}\omega + \alpha\kappa\mathring{\mathbf{A}}_1 G\gamma_0\theta = \Delta^2\omega. \tag{59}$$

2.3 Proof of Theorem 3

In proving Theorem 3, we begin with a preliminary energy identity.

LEMMA 1. *Again, with initial data* $[\omega^0, \omega^1, \theta^0] \in \boldsymbol{H}_{\kappa,\gamma}$, *we have that the component* θ *of the solution of (1)–(2) is an element of* $L^2\left(0,\infty; H^1(\Omega) \cap L^2_{\sigma+\lambda}(\Omega)\right)$; *indeed, we have the following relation* $\forall\, T > 0$:

$$-2\int_0^T \left[\eta \|\nabla\theta\|^2_{L^2(\Omega)} + \sigma \|\theta\|^2_{L^2(\Omega)} + \lambda\eta \|\theta\|^2_{L^2(\Gamma)}\right] dt = E_\gamma(T) - E_\gamma(0) \tag{60}$$

where the "energy" $E_\gamma(t)$ *is defined by*

$$E_\gamma(t) \equiv \left\|\mathring{\boldsymbol{A}}^{\frac{1}{2}}_{\kappa}\omega(t)\right\|^2_{L^2(\Omega)} + \left\|P^{\frac{1}{2}}_{\gamma}\omega_t(t)\right\|^2_{L^2(\Omega)} + \beta\|\theta(t)\|^2_{L^2(\Omega)}\,. \tag{61}$$

Proof. Starting with initial data in $D(\mathcal{A}_{\kappa,\gamma})$ which will provide $\forall$ $T>0$ that the solution $[\omega,\omega_t,\theta_t]\in C([0,T];D(\mathcal{A}_{\kappa,\gamma}))$ and $[\omega_t,\omega_{tt},\theta_t]$ $\in C([0,T];\mathbf{H}_{\kappa,\gamma})$, we have pointwise on $(0,T)$

$$\frac{d}{dt}\left\|\begin{bmatrix}\omega(t)\\ \omega_t(t)\\ \theta(t)\end{bmatrix}\right\|^2_{\mathbf{H}_{\kappa,\gamma}}=2\left(\mathcal{A}_{\kappa,\gamma}\begin{bmatrix}\omega(t)\\ \omega_t(t)\\ \theta(t)\end{bmatrix},\begin{bmatrix}\omega(t)\\ \omega_t(t)\\ \theta(t)\end{bmatrix}\right)_{\mathbf{H}_{\kappa,\gamma}},$$

and for this special choice of initial data we will have the desired equality (60) upon integration and using the fact from (12) that

$$\left(A_D(\mathbf{I}-D\gamma_0)\theta+\frac{\sigma}{\eta}\theta,\theta\right)_{L^2(\Omega)}=\left(-\Delta\theta+\frac{\sigma}{\eta}\theta,\theta\right)_{L^2(\Omega)}$$

$$=\|\nabla\theta\|^2_{L^2(\Omega)}+\frac{\sigma}{\eta}\|\theta\|^2_{L^2(\Omega)}+\lambda\|\theta\|^2_{L^2(\Gamma)}\quad\text{for }\theta\in D(A_R).\quad(62)$$

For $\sigma+\lambda>0$, The asserted L^2-regularity follows immediately from (60), inasmuch as $\{e^{\mathcal{A}_{\kappa,\gamma}t}\}_{t\geq0}$ is a contraction semigroup; for $\sigma+\lambda=0$, (60) will still yield that $\theta\in L^2\left(0,\infty;H^1(\Omega)\cap L^2_0(\Omega)\right)$, after recalling that $\int_\Omega|\nabla\theta|^2\geq C\int_\Omega\theta^2$ for all $\theta\in H^1(\Omega)\cap L^2_0(\Omega)$, and again using the contraction of the semigroup. A density argument then concludes the proof. □

REMARK 4. J. Lagnese in [3] first showed the dissipativity property (60) through a formal integration and a consequent justification through variational arguments, and the alternate proof is included here as a simple consequence of contractive semigroups.

We next derive a trace regularity result for the clamped model which does not follow from the standard Sobolev trace theory, and which is critical in our estimates of uniform decay. We note that related trace regularity results for Euler Bernoulli plates were proved in [6], and for Kirchoff plates in [5].

LEMMA 2. *With $\kappa=0$ in (1)–(2), one has the component ω of the solution $[\omega,\omega_t,\theta]$ satisfies $\Delta\omega|_\Gamma\in L^2(0,T;L^2(\Gamma))$ with the estimate*

$$\int_0^T\|\Delta\omega\|^2_{L^2(\Gamma)}\,dt\;\leq\;C\Bigg(\int_0^T\Bigg[\left\|\mathring{A}_0^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)}+$$

$$+ \left\| P_\gamma^{\frac{1}{2}} \omega_t \right\|^2_{L^2(\Omega)} + \|\nabla\theta\|^2_{L^2(\Omega)} \Bigg] dt$$

$$+ E_\gamma(T) + E_\gamma(0) \Bigg), \tag{63}$$

where C does not depend on the parameter γ.

REMARK 5. Note that the above trace estimate does not follow from the given interior regularity; this is an independently-derived trace regularity result.

Proof. If we take initial data $\left[\omega^0, \omega^1, \theta^0\right]$ in $D(\mathcal{A}^2_{0,\gamma})$, then **Theorem 2** provides that $[\omega, \omega_t, \theta]$ is a classical pointwise solution of (1)–(2). We will work to extract the desired estimate (63) in this special case–and consequently for all initial data after an extension by continuity–by multiplying the first equation of (1) by the quanity $h \cdot \nabla\omega$, where $h(x,y) \equiv [h_1(x,y), h_2(x,y)]$ is a $\left[C^2(\overline{\Omega})\right]^2$ vector field such that $h|_\Gamma = [\nu_1, \nu_2]$, and then integrating from 0 to T ; i.e. we will work with the equation

$$\int_0^T \left(\omega_{tt} - \gamma\Delta\omega_{tt} + \Delta^2\omega + \alpha\Delta\theta, h \cdot \nabla\omega\right)_{L^2(\Omega)} dt = 0. \tag{64}$$

(i) First,

$$\begin{aligned}
&\int_0^T (\omega_{tt}, h \cdot \nabla\omega)_{L^2(\Omega)}\, dt \;= \\
&= \quad (\omega_t, h \cdot \nabla\omega)_{L^2(\Omega)}\Big|_0^T - \int_0^T (\omega_t, h \cdot \nabla\omega_t)_{L^2(\Omega)}\, dt \\
&= \quad (\omega_t, h \cdot \nabla\omega)_{L^2(\Omega)}\Big|_0^T - \frac{1}{2}\int_0^T \int_\Omega div\left(\omega_t^2 h\right) dt d\Omega \\
&\qquad + \frac{1}{2}\int_0^T \int_\Omega \omega_t^2 \left[h_{1x} + h_{2y}\right] dt d\Omega \\
&= \quad (\omega_t, h \cdot \nabla\omega)_{L^2(\Omega)}\Big|_0^T + \frac{1}{2}\int_0^T \int_\Omega \omega_t^2 \left[h_{1x} + h_{2y}\right] dt d\Omega,
\end{aligned}$$

after making use of the divergence theorem and the fact that $\omega_t = 0$ on Γ.

(ii) Next

$$
\begin{aligned}
&\int_0^T (-\Delta\omega_{tt}, h\cdot\nabla\omega)_{L^2(\Omega)}\,dt \;= \\
&= \; (\nabla\omega_t, \nabla(h\cdot\nabla\omega))_{L^2(\Omega)}\Big|_0^T - \int_0^T (\nabla\omega_t, \nabla(h\cdot\nabla\omega_t))_{L^2(\Omega)}\,dt \\
&= \; (\nabla\omega_t, \nabla(h\cdot\nabla\omega))_{L^2(\Omega)}\Big|_0^T - \frac{1}{2}\int_0^T\int_\Omega div\left(|\nabla\omega_t|^2 h\right) dtd\Omega \\
&\quad - \int_0^T\int_\Omega \left[\frac{\omega_{tx}^2 h_{1x}}{2} + \frac{\omega_{ty}^2 h_{2y}}{2}\right] dtd\Omega \\
&\quad - \int_0^T\int_\Omega \left[\omega_{tx}\omega_{ty}h_{2x} + \omega_{tx}\omega_{ty}h_{1y}\right] dtd\Omega \\
&\quad + \int_0^T\int_\Omega \left[\frac{\omega_{tx}^2 h_{2y}}{2} + \frac{\omega_{ty}^2 h_{1x}}{2}\right] dtd\Omega \\
&= \; (\nabla\omega_t, h\cdot\nabla\omega)_{L^2(\Omega)}\Big|_0^T \\
&\quad + \int_0^T\int_\Omega \left[\frac{\omega_{tx}^2 h_{2y}}{2} + \frac{\omega_{ty}^2 h_{1x}}{2} - \frac{\omega_{tx}^2 h_{1x}}{2} - \frac{\omega_{ty}^2 h_{2y}}{2}\right] dtd\Omega \\
&\quad - \int_0^T\int_\Omega \left[\omega_{tx}\omega_{ty}h_{2x} + \omega_{tx}\omega_{ty}h_{1y}\right] dtd\Omega,
\end{aligned}
$$

after again using the divergence theorem and the fact that $\int_\Omega div\left(|\nabla\omega_t|^2 h\right) d\Omega = \int_\Gamma |\nabla\omega_t|^2\, d\Gamma = 0$ (as $\omega_t(t) \in H_0^2(\Omega)$).

(iii) To handle the fourth order term, we use Green's Theorem and the B.C. (5) to obtain

$$
\begin{aligned}
\int_0^T \left(\Delta^2\omega, h\cdot\nabla\omega\right)_{L^2(\Omega)} dt &= \int_0^T a\left(\omega, h\cdot\nabla\omega\right) dt \\
&\quad - \int_\Gamma \left(\Delta\omega + (1-\mu)B_1\omega\right) \frac{\partial h\cdot\nabla\omega}{\partial\nu} dtd\Gamma. \qquad (65)
\end{aligned}
$$

We note at this point that we can rewrite the first term on the

RHS of (65) as

$$\int_0^T a(\omega, h\cdot\nabla\omega)\,dt = \frac{1}{2}\int_0^T\int_\Omega h\cdot\nabla\Big[\omega_{xx}^2+\omega_{yy}^2+2\mu\omega_{xx}\omega_{yy} + 2(1-\mu)\omega_{xy}^2\Big]\,dt\,d\Omega + \mathcal{O}\left(\int_0^T \left\|\mathring{\mathbf{A}}_0^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}^2 dt\right), \tag{66}$$

where $\mathcal{O}\left(\int_0^T \left\|\mathring{\mathbf{A}}_0^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}^2 dt\right)$ denotes a series of terms which can be majorized by the $L^2(0,T;D(\mathring{\mathbf{A}}_0^{\frac{1}{2}}))$-norm of ω; in turn we have by the divergence theorem that

$$\begin{aligned}
&\int_0^T\int_\Omega h\cdot\nabla\left[\omega_{xx}^2+\omega_{yy}^2+2\mu\omega_{xx}\omega_{yy}+2(1-\mu)\omega_{xy}^2\right]dtd\Omega\\
&= \int_0^T\int_\Omega div\left\{h\left[\omega_{xx}^2+\omega_{yy}^2+2\mu\omega_{xx}\omega_{yy}+2(1-\mu)\omega_{xy}^2\right]\right\}\\
&\qquad + \mathcal{O}\left(\int_0^T \left\|\mathring{\mathbf{A}}_0^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}^2 dt\right)\\
&= \int_0^T\int_\Gamma \left[\omega_{xx}^2+\omega_{yy}^2+2\mu\omega_{xx}\omega_{yy}+2(1-\mu)\omega_{xy}^2\right]dtd\Omega\\
&\qquad + \mathcal{O}\left(\int_0^T \left\|\mathring{\mathbf{A}}_0^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}^2 dt\right).
\end{aligned} \tag{67}$$

As $\omega|_\Gamma = \left.\dfrac{\partial\omega}{\partial\nu}\right|_\Gamma = 0$, we consequently have (as reasoned in [3], Ch. 4) that

$$\omega_{xx}^2+\omega_{yy}^2+2\mu\omega_{xx}\omega_{yy}+2(1-\mu)\omega_{xy}^2 = (\Delta\omega)^2$$

on Γ; furthermore $B_1\omega = 0$, which implies that $\Delta\omega = \dfrac{\partial^2\omega}{\partial\nu^2} = \dfrac{\partial\, h\cdot\nabla\omega|_\Gamma}{\partial\nu}$. We consequently have upon the insertion of (66) and (67) into (65) that

$$\int_0^T \left(\Delta^2\omega, h\cdot\nabla\omega\right)_{L^2(\Omega)} dt = -\frac{1}{2}\int_0^T \|\Delta\omega\|^2_{L^2(\Omega)}\, dt + \mathcal{O}\left(\int_0^T \left\|\mathring{\mathbf{A}}_0^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} dt\right). \qquad (68)$$

(iv) To handle the last term on the left hand side of equation (64), we again Green's theorem and the boundary conditions (5) to obtain

$$\int_0^T (\Delta\theta, h\cdot\nabla\omega)_{L^2(\Omega)}\, dt = -\int_0^T (\nabla\theta, \nabla(h\cdot\nabla\omega))_{L^2(\Omega)}\, dt.$$

To finish the proof, we rewrite (64) by collecting the relations given by (i)–(ii) above, (68) and (iv) to thereby attain the inequality (63), upon taking norms and majorizing. □

In showing the exponential decay of the semigroup $\left\{e^{\mathcal{A}_{\kappa,\gamma}t}\right\}_{t\geq 0}$ (Theorem 3) it will suffice, as usual, to prove that there exists a time $0<T<\infty$ and a corresponding constant C_T which satisfies for all initial data in $\mathbf{H}_{\kappa,\gamma}$,

$$E_\gamma(T) \leq \xi E_\gamma(0) \text{ with } \xi < 1 \text{ and independent of } \gamma > 0. \qquad (69)$$

By a density argument, it will then be enough by Lemma 1 to show the existence of a time $0<T<\infty$ and constant C_T (independent of γ) for initial data in $\left[\omega^0, \omega^1, \theta^0\right] \in D\left(\mathcal{A}^2_{\kappa,\gamma}\right)$ such that

$$E_\gamma(T) \leq C_T \int_0^T \|\theta\|^2_{H^1(\Omega)\cap L^2_\sigma(\Omega)}\, dt, \qquad (70)$$

to which end we will proceed to work.

Because ofTheorem 2, we have for initial data $\left[\omega^0, \omega^1, \theta^0\right] \in D\left(\mathcal{A}^2_{\kappa,\gamma}\right)$ a classical pointwise solution $[\omega, \omega_t, \theta]$ of (1)–(2); we can thus multiply the first equation in (1) by $A_D^{-1}\theta$, integrate from 0 to T and obtain

$$\int_0^T \left[\left(\omega_{tt} - \gamma\Delta\omega_{tt} + \Delta^2\omega + \alpha\Delta\theta, A_D^{-1}\theta\right)_{L^2(\Omega)}\right] dt = 0. \qquad (71)$$

In dealing with this equation, we note the following:

(A.1) Using an integration by parts and the second differential equation of (1) produces

$$
\begin{aligned}
&\int_0^T \left(\omega_{tt} - \gamma\Delta\omega_{tt}, A_D^{-1}\theta\right)_{L^2(\Omega)} dt \\
&= \left(\omega_t, A_D^{-1}\theta\right)_{L^2(\Omega)}\Big|_0^T + \gamma\left(\nabla\omega_t, \nabla A_D^{-1}\theta\right)_{L^2(\Omega)}\Big|_0^T \\
&\quad - \int_0^T \left[\left(\omega_t, A_D^{-1}\theta_t\right)_{L^2(\Omega)} + \gamma\left(\nabla\omega_t, \nabla A_D^{-1}\theta_t\right)_{L^2(\Omega)}\right] dt \\
&= \alpha\beta^{-1}\int_0^T \left[\|\omega_t\|^2_{L^2(\Omega)} + \gamma\|\nabla\omega_t\|^2_{L^2(\Omega)}\right] dt \\
&\quad + \beta^{-1}\int_0^T \left(\omega_t, \eta(\mathbf{I} - D\gamma_0)\theta + \sigma A_D^{-1}\theta\right)_{L^2(\Omega)} dt \\
&\quad + \beta^{-1}\gamma\int_0^T \left(\nabla\omega_t, \nabla\left(\eta(\mathbf{I} - D\gamma_0)\theta + \sigma A_D^{-1}\theta\right)\right)_{L^2(\Omega)} dt \\
&\quad + \left(\omega_t, A_D^{-1}\theta\right)_{L^2(\Omega)}\Big|_0^T + \gamma\left(\nabla\omega_t, \nabla A_D^{-1}\theta\right)_{L^2(\Omega)}\Big|_0^T; \qquad (72)
\end{aligned}
$$

as $D\gamma_0 \in \mathcal{L}(H^1(\Omega))$ (see (15) and (18)) and A_D^{-1} is "smoothing", viz. $\left\|A_D^{-1}\theta\right\|_{H^{\frac{3}{2}+\epsilon}(\Omega)} \leq C\|\theta\|_{H^1(\Omega)}$, we have the estimates

$$\|(I - D\gamma_0)\theta\|_{L^2(\Omega)} + \left\|A_D^{-1}\theta\right\|_{L^2(\Omega)} \leq C\|\theta\|_{H^1(\Omega)}; \qquad (73)$$

$$\|\nabla(I - D\gamma_0)\theta\|_{L^2(\Omega)} + \left\|\nabla A_D^{-1}\theta\right\|_{L^2(\Omega)} \leq C\|\theta\|_{H^1(\Omega)}, \qquad (74)$$

to thereby attain

$$
\begin{aligned}
&\left|\int_0^T \left(\omega_{tt} - \gamma\Delta\omega_{tt}, A_D^{-1}\theta\right)_{L^2(\Omega)} dt \right.\\
&\left. - \alpha\beta^{-1}\int_0^T \left[\|\omega_t\|^2_{L^2(\Omega)} + \gamma\|\nabla\omega_t\|^2_{L^2(\Omega)}\right] dt\right| \leq \\
&\leq C\int_0^T \left[\|\omega_t\|_{L^2(\Omega)}\|\theta\|_{H^1(\Omega)} + \gamma\|\nabla\omega_t\|_{L^2(\Omega)}\|\theta\|_{H^1(\Omega)}\right] dt
\end{aligned}
$$

$$
\begin{aligned}
&+ C\left[E_\gamma(0) + E_\gamma(T)\right] \\
\leq\ & \epsilon \int_0^T \left[\|\omega_t\|_{L^2(\Omega)} + \gamma \|\nabla \omega_t\|_{L^2(\Omega)}\right] dt + C_\epsilon \int_0^T \|\theta\|_{H^1(\Omega)}\, dt \\
&+ C\left[E_\gamma(0) + E_\gamma(T)\right], \qquad (75)
\end{aligned}
$$

where the constant C_ϵ does not depend on $0 \leq \gamma \leq M$.

(A.2) Yet another application of Green's theorem and the characterization (7) give

$$
\begin{aligned}
\int_0^T \left(\Delta^2 \omega, A_D^{-1}\theta\right) dt \ =\ & \int_0^T a\left(\omega, A_D^{-1}\theta\right) \\
& - \int_0^T \left(\Delta\omega + (1-\mu)B_1\omega, \frac{\partial A_D^{-1}\theta}{\partial \nu}\right)_{L^2(\Gamma)} dt \qquad (76)
\end{aligned}
$$

with $a(\cdot,\cdot)$ as defined in (8), where

$$
\Delta\omega + (1-\mu)B_1\omega = \begin{cases} -\alpha\theta, & \text{if } \kappa = 1 \\ \Delta\omega \in L^2(0,T;L^2(\Gamma)), & \text{if } \kappa = 0 \end{cases}
$$

(see Lemma 2).

Therefore, estimating the right hand side of (76) yields

$$
\begin{aligned}
&\left| \int_0^T \left(\Delta^2\omega, A_D^{-1}\theta\right) dt \right| \ \leq \\
&\leq\ C\Bigg[\int_0^T \left\| \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}} \omega \right\|_{L^2(\Omega)} \|\theta\|_{H^1(\Omega)}\, dt \\
&\quad + \left[(1-\kappa)\int_0^T \|\Delta\omega\|_{L^2(\Gamma)}\, dt + \kappa \int_0^T \|\theta\|_{L^2(\Gamma)}\, dt\right] \\
&\quad \int_0^T \left\|A_D^{-1}\theta\right\|_{H^{\frac{3}{2}+\epsilon}(\Omega)} dt \Bigg] \\
&\quad \text{(by Lemma 2)} \\
&\leq\ C\int_0^T \left\| \mathring{\mathbf{A}}_\kappa^{\frac{1}{2}} \omega \right\|_{L^2(\Omega)} \|\theta\|_{H^1(\Omega)}\, dt
\end{aligned}
$$

$$
\begin{aligned}
&+C\Bigg[(1-\kappa)\int_0^T\left(\left\|\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}+\left\|P_\gamma^{\frac{1}{2}}\omega_t\right\|_{L^2(\Omega)}\right)dt\\
&+C\left[E_\gamma(0)+E_\gamma(T)\right]\Bigg]\int_0^T\left\|A_D^{-1}\theta\right\|_{H^{\frac{3}{2}+\epsilon}(\Omega)}dt\\
&+\alpha\kappa C\int_0^T\|\theta\|_{L^2(\Omega)}dt\cdot\int_0^T\left\|A_D^{-1}\theta\right\|_{H^{\frac{3}{2}+\epsilon}(\Omega)}dt\\
\leq\ &\epsilon\int_0^T\left[\left\|\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}^2+\left\|P_\gamma^{\frac{1}{2}}\omega_t\right\|_{L^2(\Omega)}\right]dt\\
&+\varepsilon\left[E_\gamma(0)+E_\gamma(T)\right]+C_\epsilon\int_0^T\|\theta\|_{H^1(\Omega)}dt, \qquad (77)
\end{aligned}
$$

where we have once more used trace theory and elliptic regularity.

(A.3) Finally, for the last term of (71)

$$
\begin{aligned}
&\alpha\int_0^T\left(A_D(\mathbf{I}-D\gamma_0)\theta,A_D^{-1}\theta\right)_{L^2(\Omega)}dt\ =\\
&=\ \alpha\int_0^T\left[\|\theta\|_{L^2(\Omega)}^2-(D\gamma_0\theta,\theta)_{L^2(\Omega)}\right]dt\\
&\leq\ C\int_0^T\|\theta\|_{H^1(\Omega)}^2. \qquad (78)
\end{aligned}
$$

Combining (71), (75), (77) and (78) thus results in the following:

(A.4) For $\epsilon>0$ small enough there exists a constant $C>0$ such that the solution $[\omega,\omega_t,\theta]$ of (1)–(2) satisfies

$$
\begin{aligned}
&\left(\frac{\alpha}{\beta}-2\epsilon\right)\int_0^T\left[\|\omega_t\|_{L^2(\Omega)}^2+\gamma\|\nabla\omega_t\|_{L^2(\Omega)}^2\right]dt\\
&\leq\ C\left[\int_0^T\|\nabla\theta\|_{L^2(\Omega)}^2+\|\theta\|_{L^2(\Omega)}^2dt\right.\\
&\qquad\left.+E_\gamma(T)+E_\gamma(0)\right]+\epsilon\int_0^T\left\|\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega\right\|_{L^2(\Omega)}^2dt, \qquad (79)
\end{aligned}
$$

where the noncrucial dependence of C upon ϵ has not been noted.

To majorize the norm of the component ω, we multiply (33) by ω and integrate from 0 to T to obtain (after accounting for the boundary conditions)

$$\left(P_\gamma^{\frac{1}{2}}\omega_t, P_\gamma^{\frac{1}{2}}\omega\right)_{L^2(\Omega)}\Big|_0^T - \int_0^T \left\|P_\gamma^{\frac{1}{2}}\omega_t\right\|^2_{L^2(\Omega)} dt$$

$$= -\int_0^T \left\|\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} dt - \alpha\kappa \int_0^T \left(\theta, \frac{\partial\omega}{\partial\nu}\right)_{L^2(\Gamma)} dt$$

$$+ \alpha \int_0^T (\nabla\theta, \nabla\omega)_{L^2(\Omega)}\, dt; \tag{80}$$

Since by the trace theorem

$$\left|\left(\theta, \frac{\partial\omega}{\partial\nu}\right)_{L^2(\Gamma)}\right| + \left|(\nabla\theta, \nabla\omega)_{L^2(\Omega)}\right| \le$$

$$\le C\left[\|\theta\|_{H^{\frac{1}{2}}(\Gamma)}\|\omega\|_{H^{\frac{3}{2}}(\Gamma)} + \|\theta\|_{H^1(\Omega)}\|\omega\|_{H^1(\Omega)}\right]$$

$$\le C\|\theta\|_{H^1(\Omega)}\|\omega\|_{H^1(\Omega)} \le \epsilon\left\|\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} + C_\epsilon\|\theta\|^2_{H^1(\Omega)}, \tag{81}$$

we thus arrive at

(A.5) There exists a constant $C > 0$ such that for $\epsilon > 0$ small enough, the the solution $[\omega, \omega_t, \theta]$ of (1)–(2) satisfies

$$(1-\epsilon)\int_0^T \left\|\mathring{\mathbf{A}}_\kappa^{\frac{1}{2}}\omega\right\|^2_{L^2(\Omega)} dt \le$$

$$\le C\int_0^T \left[\|\omega_t\|^2_{L^2(\Omega)} + \gamma\|\nabla\omega_t\|^2_{L^2(\Omega)}\right] dt$$

$$+ C\left(\int_0^T \|\theta\|^2_{H^1(\Omega)}\, dt + E_\gamma(T) + E_\gamma(0)\right), \tag{82}$$

where the noncrucial dependence of C upon ϵ has not been noted.

Thus, if ϵ of (A.4) and (A.5) is small enough, we then have, upon combining (A.4) and (A.5), the existence of a constant C (independent of γ) such that

$$\int_0^T \left[\left\| \mathring{\mathbf{A}}^{\frac{1}{2}} \omega \right\|^2_{L^2(\Omega)} + \|\omega_t\|^2_{L^2(\Omega)} + \gamma \|\nabla \omega_t\|^2_{L^2(\Omega)} \right] dt \le$$

$$\le C \left[E_\gamma(T) + E_\gamma(0) + \int_0^T \left[\|\nabla\theta\|^2_{L^2(\Omega)} + \|\theta\|^2_{L^2(\Omega)} \right] dt \right]. \quad (83)$$

To conclude the proof of Theorem 3, we apply the relation (60) and its inherent dissipativity property (that is, $E_\gamma(T) \le E_\gamma(t)$ $\forall$ $0 \le t \le T$) to (83) to finally attain the sought-after inequality; namely, for $T > 2C$ (with C independent of $\gamma > 0$),

$$E_\gamma(T) < \frac{3C+1}{T-2C} \int_0^T \|\theta\|^2_{H^1(\Omega)\cap L^2_\sigma(\Omega)} dt \quad (84)$$

which, as noted above, will imply (69).

Remark 6. We note that our proof can easily be adapted to the situation where boundary conditions are partially clamped, as was considered recently by Liu and Zheng in [8] (albeit only in the case $\gamma \equiv 0$); that is, with $\Gamma = \Gamma_0 \cup \Gamma_1$ and $\overline{\Gamma}_0 \cap \overline{\Gamma}_1 = \varnothing$, we can show, by the same direct method employed for Theorem 3, the uniform decay of solutions of (1) with $\gamma \ge 0$ and the boundary conditions

$$\begin{cases} \omega = \dfrac{\partial\omega}{\partial\nu} = 0 \text{ on } (0,\infty)\times\Gamma_0 \\ \omega = \Delta\omega + (1-\mu)B_1\omega + \alpha\theta = 0 \text{ on } (0,\infty)\times\Gamma_1. \end{cases} \quad (85)$$

Indeed, all the arguments presented above will be the same, the sole exception being that the requisite regularity lemma is applied only on a portion of the boundary. Thus, instead of Lemma 2, we will use the following one.

Lemma 3. *The component ω of the unique solution $[\omega, \omega_t, \theta]$ to (1), (85) satisfies $\Delta\omega|_\Gamma \in L^2(0,T;L^2(\Gamma_0))$ with the estimate*

$$\int_0^T \|\Delta\omega\|^2_{L^2(\Gamma_0)} dt \le$$

$$\le C \left(\int_0^T \left[a(\omega,\omega) + \|\nabla\theta\|^2_{L^2(\Omega)} \right] dt + E(T) + E(0) \right), \quad (86)$$

where $E_\gamma(t) \equiv a(\omega(t),\omega(t)) + \left\| P_\gamma^{\frac{1}{2}} \omega_t(t) \right\|^2_{L^2(\Omega)} + \beta \|\theta(t)\|^2_{L^2(\Omega)}$, *and* $a(\cdot,\cdot)$ *is as defined in (8).*

Proof. The same as in Lemma 2, the only difference being that we make use of a $\left[C^2(\overline{\Omega})\right]^2$ vector field h which satisfies $h \equiv \nu$ on Γ_0, and $h \equiv 0$ on Γ_1. □

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