SOME REMARKS ON CATEGORIES WITH CONSTANT MAPS AND MONOIDAL CLOSEDNESS (*)

by M. CRISTINA PEDICCHIO and F. ROSSI (**)

SOMMARIO. - *In questo articolo si risponde alla seguente questione: «La riflessione di una struttura monoidale chiusa (canonica) su una categoria topologica è ancora canonica?». Vengono analizzate varie situazioni possibili ed assegnati mediante la teoria dei funtori topologici e semi-topologici, numerosi esempi atti ad illustrarle.*

SUMMARY. - *We study the «canonicity» of monoidal closed structures on categories with constant maps. The main results regard monoidal closed structures obtained by a «normal reflective embedding» on opportune reflective subcategories.*

Introduction. There is a great difference, concerning monoidal closed structures, between algebraic and topological categories. In the algebraic case we have at most one monoidal closed structure $(-\square-, I, r, l, a, [-,-])$, such that $\mathcal{U}(A \square B) \cong \mathcal{U}A \times \mathcal{U}B$, where $\mathcal{U}$ is the forgetful functor on **Set**, while in the topological case there is at least one structure of this kind [15], [12], and in many cases even a proper class [4], [5], [6]. Observe that, in the latter context, the forgetful functor $\mathcal{U}$ is strict monoidal with respect to the cartesian closed structure of **Set**, and all the natural isomorphisms of monoidal closed structure have canonical underlying maps. In this paper, we shall always refer to such a structure as to a *canonical structure*. In [10] it has been proved that there exists a large class

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(**) Indirizzo degli Autori: Istituto di Matematica dell'Università degli Studi di Trieste - Piazzale Europa, 1 - 34100 Trieste.

of initially structured (in the sense of Nel and Wyler [11], [16]) but non-topological categories, where *all* the monoidal closed structures are canonical. Other examples are given in [12], [13] and [1].

Our present aim is to investigate the following problem: *Let us consider a symmetric monoidal closed (necessarily canonical) structure on any topological category* **B**. *Let* **A** *be a full reflective subcategory of* **B** *such that the embedding* E *admits an enrichment to a normal reflective one; is the reflected structure still canonical?*

First of all, we give an example of a *non-canonical* reflected structure. Then, in the more general context of concrete categories with constant maps, we give a sufficient condition in order that the reflected structure *is canonical* and discuss it in the topological case. Finally we show by another example that the previous condition *is not necessary.*

1. **Preliminaries.** The aim of this Section is to recall some results about monoidal closed structures on concrete categories with constant maps. For more details, see [10].

Let **A** be a *concrete* category (i.e. there exists a faithful functor $\mathcal{U}:\mathbf{A}\to\mathbf{Set}$) with the following proprieties:

a) *for every constant map* $f:\mathcal{U}A\to\mathcal{U}B$ *there exists an* **A**-*morphism* $\bar{f}:A\to B$ *with* $\mathcal{U}(\bar{f})=f$;

b) $\mathcal{U}$ *transports structures;*

c) *there exists an* **A**-*object* A *with* $\mathrm{card}(\mathcal{U}A)\geqslant 2$.

Let $(-\square -, I, r, l, a, [-,-])$ be a monoidal closed structure on **A**.

PROPOSITION 1.1: *The structure* $(-\square -, I, r, l, a, [-,-])$ *is (up to natural isomorphisms) as follows:*

1) $\mathcal{U}I=\mathbf{1}$, *terminal object of* **Set**; I *is also terminal in* **A** *and it is a representing object for* $\mathcal{U}$;

2) $\mathcal{U}A\times\mathcal{U}B\subseteq\mathcal{U}(A\square B)$, *where the inclusion is natural in* A,B;

3) $\mathcal{U}[B,C]=\mathbf{A}(B,C)$;

4) *For every* $u,v:A\square B\to C$ *such that* $\mathcal{U}u\big|_{\mathcal{U}A\times\mathcal{U}B}=\mathcal{U}v\big|_{\mathcal{U}A\times\mathcal{U}B}$, *it follows* $u=v$;

5) *If* $\pi:\mathbf{A}(A\square B,C)\cong\mathbf{A}(A,[B,C])$ *is the adjunction, then* $\mathcal{U}[(\mathcal{U}\pi(f))(x)]=(y\mapsto\mathcal{U}f(x,y))$ *and* $(\mathcal{U}\pi^{-1}(g))(x,y)=(\mathcal{U}g)(x)(y)$, *for every* $f:A\square B\to C$, $g:A\to[B,C]$, $(x,y)\in\mathcal{U}A\times\mathcal{U}B$;

6) $\mathcal{U}I\times\mathcal{U}A=\mathcal{U}(I\square A)$, $\mathcal{U}A\times\mathcal{U}I=\mathcal{U}(A\square I)$, $(\mathcal{U}l)(*,x)=x$,

$(\mathcal{U}r)(x,*) = x$, $\mathcal{U}a \big|_{(\mathcal{U}A \times \mathcal{U}B) \times \mathcal{U}C}((x,y),z) = (x,(y,z))$. *If the monoidal structure is symmetric, then* $c \big|_{\mathcal{U}A \times \mathcal{U}B}(x,y) = (y,x)$.

DEFINITION 1.2: *Let* **A** *be a concrete category. A monoidal closed structure on* **A** *such that verifies the conditions 1), 3), 5), 6) of* 1.1 *and, instead of 2), the stronger:*

2') $\mathcal{U}A \times \mathcal{U}B = \mathcal{U}(A \square B)$ *for all* $A, B \in \mathbf{A}$,
is called canonical.

Of course, for every canonical monoidal closed structure on **A**, the natural isomorphisms $a, r, l, (c)$ and the adjunction π have the obvious underlying bijections.

THEOREM 1.3: *If* **A** *is a concrete category that verifies the previous conditions a), b), c) and also:*

d) for every $X \subseteq \mathcal{U}A$ *there exists a morphism* $j: B \to A$ *such that* $\mathcal{U}B = X$ *and* $\mathcal{U}j$ *is the inclusion;*

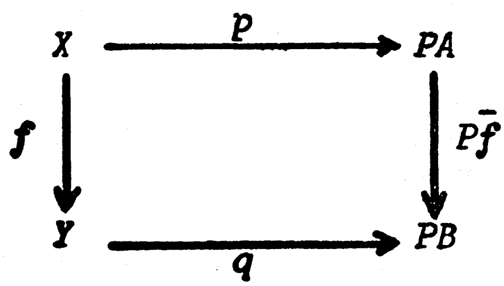
e) $\mathcal{U}$ *preserves epimorphisms,*
then all monoidal closed structures on **A** *are canonical (up to natural isomorphisms).*

2. Reflected structures. Examples.

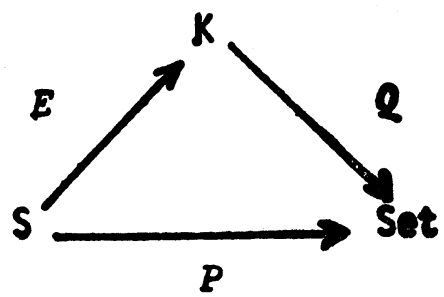
A class of categories satisfying Theorem 1.3 is given in [10, Remark 1.5]. Now, let **B** be a concrete category which verifies the hypotheses of 1.1, and let **A** be a full (replete) and reflective subcategory of **B** with embedding functor E. Furthermore, let $(-\square-, I, r, l, a, c, [-,-])$ be a canonical symmetric monoidal closed structure on **B**, such that E admits an enrichment to a normal reflective embedding (in the sense of [2]). Our aim is to study the canonicity of the reflected structure in such a situation, that we shall call *basic situation*.

First we give an example of a non-canonical reflected structure. Let **S** be the category of semi-lattices and preserving binary suprema maps; and let $P: \mathbf{S} \to \mathbf{Set}$ be the natural forgetful functor. Since **S** is an equational variety of algebras for a commutative theory, then **S** admits a symmetric monoidal closed structure $(-\square_S-, I, r, l, a, c, [-,-]_S)$, where the tensor product represents bimorphisms (see [9] and [3]). Since P is monadic over **Set**, then P is semi-topological in the sense of Tholen [14]; furthermore P is functional in the sense of Porst-Wischnewsky [12] and $-\square_S-$ and $[-,-]_S$

are, respectively, the P-semifinal tensor product (generated by the cartesian closed structure on **Set**) and the semiinitial Hom-functor on **S** (see [12]). Denote by $Q = \mathrm{Quot}(P)$ the set $\{(e,A) : (e,A)$ is a P-quotient, $e: X \to PA\}$ (see [14] p. 56), and by **K** the category whose objects are elements of Q and whose morphisms are pairs $(f,\bar{f}) : (p,A) \to (q,B)$ with $f: X \to Y$ in **Set** and $\bar{f}: A \to B$ in **S** such that



is commutative. From [14] it follows that P admits a factorization



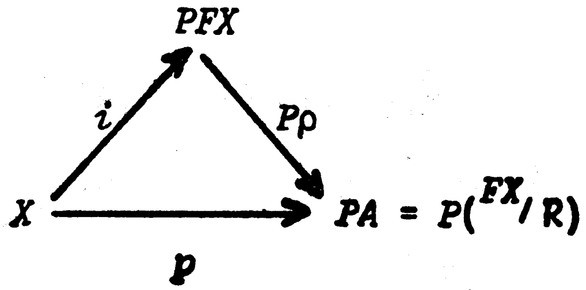
where $\mathcal{Q}$ is (properly) topological, and E is a full reflective embedding; furthermore $\mathcal{Q}(p,A) = X, E(A) = (1_{PA}, A)$ and the reflection $R: \mathbf{K} \to \mathbf{S}$ is such that $R(p,A) = A$. So we can consider **S** as a full, reflective subcategory of **K**. $\mathcal{Q}$ is topological, then **K** is symmetric monoidal closed with structure $(-\square_{\mathbf{K}}-, I', r', l', a', c', [-,-]_{\mathbf{K}})$, where $-\square_{\mathbf{K}}-$ is the $\mathcal{Q}$-(semi) final tensor product, and $[-,-]_{\mathbf{K}}$ is the (semi) initial Hom-functor.

PROPOSITION 2.1. *i)* **K** *is topological in the sense of Herrlich* [7], *so* **K** *verifies the hypotheses of* 1.3;

ii) the embedding E is a normal reflective embedding with respect to $(-\square_{\mathbf{K}}-, I', r', l', a', c', [-,-]_{\mathbf{K}})$ *and to* $(-\square_{\mathbf{S}}-, I, r, l, a, c, [-,-]_{\mathbf{S}})$;

iii) $(-\square_{\mathbf{S}}-, I, r, l, a, c, [-,-]_{\mathbf{S}})$ *is not canonical.*

Proof. Let us consider any P-quotient $p: X \to PA$ as a composite



where FX is the free semi-lattice on $X, \mathfrak{R}$ is a congruence on FX, and ρ is the canonical epimorphism; since $F(1) = 1$, then $\mathbf{K}$ is well-fibred. $\mathcal{Q}$ is amnestic; if $\psi: Y \cong X$ and $(p, A) \in Q$, then also $(p \cdot \psi, A) \in Q$, so $\mathcal{Q}$ is transportable and *i)* follows.

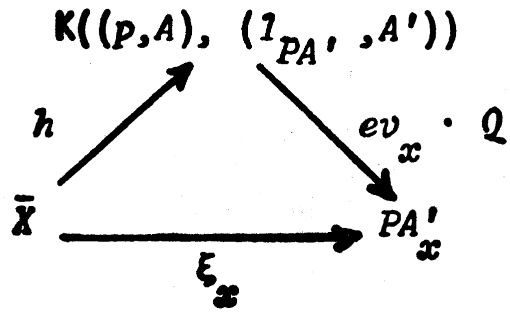
ii) Consider $(p, A) \in \mathbf{K}$ and $(1_{PA'}, A') \in \mathbf{S}$; we shall prove that $[(p, A), (1_{PA'}, A')]_{\mathbf{K}} \in \mathbf{S}$.

Denote by $\hat{\mathbf{K}}((p, A), (1_{PA'}, A'))$ the set $\mathbf{K}((p, A), (1_{PA'}, A'))$ with the semi-lattice structure defined by $(f, \bar{f}) \vee (\varphi, \bar{\varphi}) = (f \vee \varphi, \bar{f} \vee \bar{\varphi})$, where $\vee$ is the pointwise composition. The $\mathcal{Q}$-cone

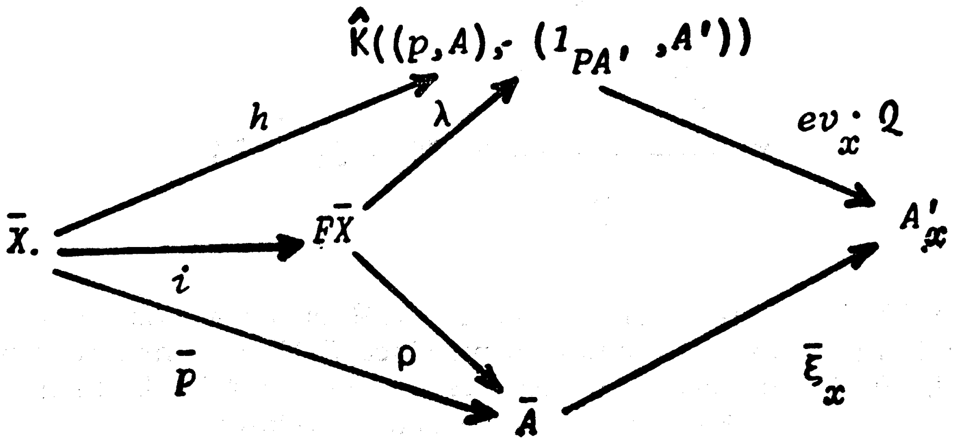
$$\mathbf{K}((p, A), (1_{PA'}, A')) \xrightarrow{\mathcal{Q}} \mathbf{Set}(X, PA') \xrightarrow{ev_x} \mathcal{Q}(1_{PA'_x}, A'_x) = PA'_x$$

where $A'_x = A'$ and ev_x is the evaluation in x, for any $x \in X$, admits a lifting in $\mathbf{K}$, for $ev_x \cdot \mathcal{Q}: \hat{\mathbf{K}}((p, A), (1_{PA'}, A')) \to PA'_x$ is a $\mathbf{S}$-morphism (for any x). Let us prove that such a lifting is initial in $\mathbf{K}$.

Consider a $\mathcal{Q}$-cone in $\mathbf{K}$, $(\xi_x, \bar{\xi}_x): (\bar{p}, \bar{A}) \to (1_{PA'_x}, A'_x)$, such that the following diagram



is commutative, for any $x \in X$. Now it suffices to give a morphism $\tau: \bar{A} \to \hat{\mathbf{K}}((p, A), (1_{PA'}, A'))$ in $\mathbf{S}$, such that $P\tau \cdot \bar{p} = h$. Since $(\bar{p}, \bar{A}) \in Q$, $\bar{A}$ is a quotient $F\bar{X}/\mathfrak{R}$ with respect to a congruence $\mathfrak{R}$ on the free semi-lattice $F\bar{X}$, and $\bar{p} = \rho \cdot i$, where $\rho: F\bar{X} \to \bar{A}$ is the canonical epimorphism and $i: \bar{X} \to F\bar{X}$ is the inclusion (we omit the symbol P for the sake of brevity). Then, in the following diagram



there exists an unique **S**-morphism $\lambda: F\bar{X} \to \hat{\mathbf{K}}((p,A), (1_{PA'}, A'))$ with $\lambda \cdot i = h$; furthermore $\bar{\xi}_x \cdot \rho = ev_x \cdot \mathcal{Q} \cdot \lambda$, for any $x \in X$.

By the previous equality and since $p: X \to PA$ is a P-epimorphism, it is possible to define a map $\tau: \bar{A} \to \hat{\mathbf{K}}((p,A), (1_{PA'}, A'))$ putting $\tau(\rho(y)) = \lambda(y)$, for any $y \in F\bar{X}$; since λ and ρ are **S**-morphism, then τ is a **S**-morphism.

From the definition of internal Hom in **K**, it follows that

$$[(p,A), (1_{PA'}, A')]_{\mathbf{K}} \cong (1_{\mathbf{K}((p,A),(1_{PA'},A'))}, \hat{\mathbf{K}}((p,A), (1_{PA'}, A')));$$

so it is in **S**. Since the map $\psi: \mathbf{K}((p,A), (1_{PA'}, A')) \to \mathbf{S}(A, A')$ defined by $\psi(f, \bar{f}) = \bar{f}$ is an isomorphism between $\hat{\mathbf{K}}((p,A), (1_{PA'}, A'))$ and $[A, A']_{\mathbf{S}}$, then $R[(p,A), (1_{PA'}, A')]_{\mathbf{K}} \cong [(p,A), (1_{PA'}, A')]_{\mathbf{K}} \cong [A, A']_{\mathbf{S}}$ and $R(A_1 \square_{\mathbf{K}} A_2) \cong A_1 \square_{\mathbf{S}} A_2$, for any $A_1, A_2 \in \mathbf{S}$.

ii) follows from the reflection theorem of Day [2] by easy calculations.

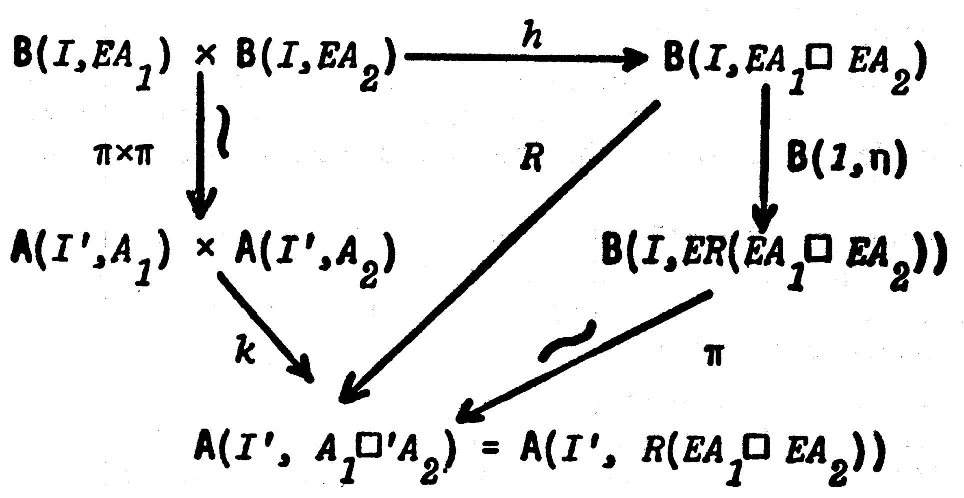
iii) See [10] Section 2. ■

Now, we will give a Proposition to assure the canonicity of the reflected structure, in the basic situation. Let **C** be a concrete category and let $\mathcal{U}: \mathbf{C} \to \mathbf{Set}$ be the forgetful functor.

DEFINITION 2.2. A **C**-morphism f is called $epi_{\mathcal{U}}$ iff $\mathcal{U}f$ is epimorphism.

PROPOSITION 2.3. *In the basic situation, if* **A** *is* $epi_{\mathcal{U}}$*-reflective in* **B***, then the reflected structure is canonical.*

Proof. Denote by $(-\Box'-, I', r', l', a', c', [-,-]')$ the symmetric monoidal closed structure obtained reflecting the **B**-structure $(-\Box-, I, r, l, a, c, [-,-])$ on **A**. By Day's reflection theorem, the following diagram



is commutative, for any $A_1, A_2 \in$ **A**, where $h(f,g) = (f \Box g) \cdot l^{-1}, f: I \to$ $\to EA_1, g: I \to EA_2$, (the same for k in **A**) and η is the unit the adjunction π. **B** verifies the hypotheses of 1.1, so I is terminal in **B** and $\mathcal{U}(-) \cong \mathbf{B}(I,-)$; then $\mathbf{B}(1,\eta)$ is epimorphism and, since h is bijective, k is epimorphism. Since **A** is reflective in **B**, $I \in$ **A**, so $I = I'$; if card$(\mathcal{U}' A) \leqslant 1$, for any $A \in$ **A**, $(\mathcal{U}' = \mathcal{U} \cdot E)$, then k is an isomorphism and the structure is canonical; if there exists an object $A \in$ **A** with card$(\mathcal{U}' A) \geqslant 2$, **A** verifies the hypotheses of 1.1 with respect to $\mathcal{U}'$; so k is injective and then k is bijective. By 1.1 it follows that the structure is canonical. ■

EXAMPLES. Many examples of the previous situation are determined by topological categories **B**. Let **B** be topological and let $(-\Box_{\mathbf{B}}-, I, r, l, a, c, [-,-]_{\mathbf{B}})$ be the symmetric monoidal closed structure on **B**, where $-\Box_{\mathbf{B}}-$ and $[-,-]_{\mathbf{B}}$ are, respectively, the $\mathcal{U}$-final tensor and the $\mathcal{U}$-initial Hom (see [15] section 3, example 3; $\mathcal{U}: \mathbf{B} \to \mathbf{Set}$); it is easily seen that $[A,B]_{\mathbf{B}}$ is an extremal subobject of $B^{\mathcal{U}A}$ (power of B in **B**). Then, the following hold.

PROPOSITION 2.4. *For any full, (replete) and epireflective subcategory* **A** *of* **B**, *the embedding E admits an enrichment to a normal reflective embedding with respect to the structure* $(-\Box_{\mathbf{B}}-, I, r, l, a, c, [-,-]_{\mathbf{B}})$; *furthermore* **A** *verifies 2.3, so the reflected structure is canonical.*

Proof. Day's theorem can be applied for **A** is epireflective. The result follows from classical properties of topological categories. ∎

REMARK 2.5. All the categories **A** of 2.4 are initially structured, in the sense of Nel [11], and the reflected structures are just the structures introduced by Porst-Wischnewsky [12] and by Činčura [1]. See also [11], [7].

Condition 2.3 is not necessary; in fact, it is possible to give the following example of a basic situation where **A** is reflective, but not $\text{epi}_{\mathcal{U}}$-reflective in **B** and the reflected structure is canonical.

Denote by $\mathbf{B} = {}^q\mathbf{Met}$ the category of quasi-metric spaces and non-expansive maps. The objects of ${}^q\mathbf{Met}$ are pairs (X, d_X), where X is a set and $d_X : X \times X \to [0, \infty]$ is a map satisfying the natural pseudo-metric properties.

The morphisms of ${}^q\mathbf{Met}$ are maps $f : (X, d_X) \to (Y, d_Y)$ such that $d_Y(f(x_1), f(x_2)) \leqslant d_X(x_1, x_2)$, for any $x_1, x_2 \in X$ (see [8] p. 321). Let $\mathbf{Sep}\text{-}{}^q\mathbf{Met}$ be the full subcategory of ${}^q\mathbf{Met}$ of separated spaces $(d(x, y) = 0 \Rightarrow x = y)$ and let $\mathbf{A} = \mathbf{C}\text{-}{}^q\mathbf{Met}$ be the subcategory of Cauchy-complete separated q-metric spaces.

It is easily seen that **A** is reflective in **B**, but not $\text{epi}_{\mathcal{U}}$-reflective; in fact ${}^q\mathbf{Met}$ is a topological category with natural forgetful functor $\mathcal{U} : {}^q\mathbf{Met} \to \mathbf{Set}$ and, if $(X, d_X) \in \mathbf{Sep}\text{-}{}^q\mathbf{Met}$, the inclusion into its Cauchy-completion (Y, d_Y) is not, generally, surjective (it is dense with respect to the topology induced by d_Y, see [8]).

Let $(-\Box_{\mathbf{B}}-, I, r, l, a, c, [-,-]_{\mathbf{B}})$ be the structure on **B**, where $-\Box_{\mathbf{B}}-$ and $[-,-]_{\mathbf{B}}$ are the $\mathcal{U}$-final tensor product and the $\mathcal{U}$-initial Hom, respectively.

PROPOSITION 2.6. *i)* $E : \mathbf{A} \to \mathbf{B}$ *admits an enrichment to a normal reflective embedding;*

ii) The reflected structure is canonical for it is the restriction on **A** *of the structure* $(-\Box_{\mathbf{B}}-, I, r, l, a, c, [-,-]_{\mathbf{B}})$.

Proof. i) Let $(Y, d_Y) \in \mathbf{C}\text{-}{}^q\mathbf{Met}$; we shall prove that $[(X, d_X), (Y, d_Y)]_{\mathbf{B}} \in \mathbf{C}\text{-}{}^q\mathbf{Met}$, for any $(X, d_X) \in {}^q\mathbf{Met}$. By definition, $[(X, d_X), (Y, d_Y)]_{\mathbf{B}}$ is the set $\mathbf{B}((X, d_X), (Y, d_Y))$-that we shall denote by $\mathbf{B}(X, Y)$-with the following q-metric:

$$d_{\mathbf{B}(X,Y)}(f, g) = \sup_{x \in X}\{d_Y(f(x), g(x))\}.$$

Then, $(\mathbf{B}(X, Y), d_{\mathbf{B}(X,Y)}) \in \mathbf{Sep}\text{-}{}^q\mathbf{Met}$; furthermore, if $\{f_n\}_{n \in N}$ is a Cauchy-sequence in $(\mathbf{B}(X, Y), d_{\mathbf{B}(X,Y)})$, then $\{f_n(x)\}_{n \in N}$ is an uniformly Cauchy-sequence in (Y, d_Y). Since (Y, d_Y) is complete, $\{f_n(x)\}_{n \in N}$ is pointwise convergent to a map $f: X \to Y$. Defining a «genuine metric» d'_Y on Y by $d'_Y(y_1, y_2) = \min(1, d_Y(y_1, y_2))$, it is easily seen that $\{f_n(x)\}_{n \in N}$ uniformly converges to $f(x)$ in (Y, d_Y), for such a convergence is uniforme in (Y, d'_Y); so f is not expansive and $\{f_n\}_{n \in N}$ converges to f in $(\mathbf{B}(X, Y), d_{\mathbf{B}(X,Y)})$.

i) Follows by Day's theorem.

ii) For any $(X, d_X), (Y, d_Y) \in {}^q\mathbf{Met}$, $(X, d_X) \square_{\mathbf{B}} (Y, d_Y) = (X \times Y, d_{\square_{\mathbf{B}}})$, where $d_{\square_{\mathbf{B}}}((x, y), (x_1, y_1)) = sup_f\{d_C(f(x, y), f(x_1, y_1))\}$, for any $(C, d_C) \in$ $\in {}^q\mathbf{Met}$ and for any $f: X \times Y \to C$ such that $f(x, —)$ and $f(—, y)$ are non-expansive for every $x \in X$ and $y \in Y$. Since $p_X: X \times Y \to X$ and $p_Y: X \times Y \to Y$ satisfy such a condition, then $d_{\square_{\mathbf{B}}}((x, y), (x_1, y_1)) \geqslant$ $\geqslant d_{prod}((x, y), (x_1, y_1))$, where d_{prod} is the q-metric of the ${}^q\mathbf{Met}$-product. Since

$$d_C(f(x, y), f(x_1, y_1)) \leqslant d_C(f(x, y), f(x, y_1)) +$$
$$+ d_C(f(x, y_1), f(x_1, y_1)) \leqslant d_Y(y, y_1) +$$
$$+ d_X(x, x_1) \leqslant 2\, max\, (d_X(x, x_1), d_Y(y, y_1)) =$$
$$= 2\, d_{prod}((x, y), (x_1, y_1));\ \text{then}\ d_{prod}((x, y), (x_1, y_1)) \leqslant$$
$$\leqslant d_{\square_{\mathbf{B}}}((x, y), (x_1, y_1)) \leqslant 2\, d_{prod}((x, y), (x_1, y_1)),\ \text{for any}\ (x, y), (x_1, y_1).$$

If $(X, d_X), (Y, d_Y) \in \mathbf{C}\text{-}{}^q\mathbf{Met}$, then $(X, d_X) \times (Y, d_Y) \in \mathbf{C}\text{-}{}^q\mathbf{Met}$, so *ii)* follows. ■

REFERENCES

[1] J. ČINČURA, *On a tensor product in initially structured categories*. Math. Slovaca, 29 (1979), 245-255.

[2] B. DAY, *A reflection theorem for closed categories*. J. Pure and Appl. Algebra, 2 (1972), 1-11.

[3] S. EILENBERG, G. M. KELLY, *Closed categories*. Proc. Conf. Cat. Alg. (La Jolla 1965), 421-562.

[4] G. GREVE, *How many monoidal closed structures are there in Top?*. Arch. Math. (Basel), 34 (1980), 538-539.

[5] G. GREVE, *Rigid spaces and monoidal closedness*. Lecture Notes in Math., 915 (1982), 105-111.

[6] G. GREVE, *General construction of monoidal closed structures in topological, uniform and nearness spaces*. Lecture Notes in Math., 962 (1982), 100-114.

[7] H. HERRLICH, *Categorical topology 1971-1981.* General topology and its relations to modern analysis and algebra V, Proc. 5th Prague Topol. Symp. 1981 (1983), 279-383.

[8] R. E. HOFFMANN, *Topological functors admitting generalized Cauchy-completions.* Lecture Notes in Math., 540 (1976), 286-344.

[9] F. J. LINTON, *Autonomous equational categories.* J. Math. Mech., 15 (1966), 637-642.

[10] S. LOGAR, F. ROSSI, *Monoidal closed structures on categories with constant maps,* to appear in J. Austral. Math. Soc. (Series A), 38 (1985).

[11] L. D. NEL, *Initially structured categories and cartesian closedness.* Can. J. Math., 27 (1975), 1361-1377.

[12] H. PORST, M. WISCHNEWSKY, *Every topological category is convenient for Gelfand duality.* Manuscripta Math., 25 (1978), 169-204.

[13] H. PORST, M. WISCHNEWSKY, *Existence and applications of monoidally closed structures in topological categories.* Lecture Notes in Math., 719 (1979), 277-292.

[14] W. THOLEN, *Semi-topological functors I.* J. Pure and Appl. Algebra, 15 (1979), 53-73.

[15] M. WISCHNEWSKY, *Aspects of categorical Algebra in initialstructure categories.* Cahiers Topo. Géo. diff. XV, 4 (1974), 419-444.

[16] O. WYLER, *Function spaces in topological categories.* Lecture Notes in Math., 719 (1979), 411-420.